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CONSIDERING PRICE COMPETITION IN ENTERPRISE DECISIONS UNDER A LOW CARBON ECONOMY BACKGROUND

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Abstract

Based on the carbon dioxide emission-dependent market represented by the traditional energy, electricity, petroleum and chemical industries, this paper studies the optimal price and competitive decision in the duopoly competition market under the constraints of three carbon emission reduction policies: carbon cap, carbon cap-and-trade and carbon tax, and verifies the relevant conclusions through numerical analysis. The results show that because of the competition between the two oligarchic markets, enterprises require a greater capacity to reduce the market price of products and gain a competitive advantage and that for the government, when the carbon cap is set and the market competition is not fierce, allowing the internal trading of carbon emission rights will create the highest profits for suppliers, consumers and society as a whole. From the perspective of energy saving and emission reduction, carbon tax policy is more conducive to encouraging suppliers to use low carbon technology.

Keywords: carbon cap; carbon cap-and-trade; carbon tax; enterprise decision, price competition

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1. Introduction

Since the industrial revolution, represented by the extensive use of steam engines, fossil fuels such as coal, oil and natural gas have begun to be widely used as the energy base. While they provide tremendous power and benefits for human production and life, they inevitably produce tremendous environmental side effects. Especially in recent years, with the acceleration of human industrialization and urbanization, the combustion of coal, oil and natural gas has been intensified, the impact on the global environment and climate has been further intensified, the extreme weather has increased, and the trend of global warming has accelerated significantly (Dahlman, 2020; EIA, 2019). The Fifth Assessment Report of the IPCC (United Nations Intergovernmental Panel on Climate Change) - Climate Change 2014 pointed out from 2003 to 2012, the global average surface temperature increased by 0.78°C (0.72-0.85°C) compared with the average temperature from 1850 to 1990. From 1901 to 2010, the global average sea level has risen by approximately 19 cm. Since the 1950s, more than half of global warming has been caused by greenhouse gas emissions such as carbon dioxide and methane caused by human activities (IPCC, 2014).

The ongoing adverse effects of global climate change have attracted increasing attention from governments and people of all countries. Governments around the world actively take action to formulate policies and set carbon emission reduction targets. Currently, the major carbon emission reduction policies implemented worldwide are as follows: carbon cap policy, carbon emissions tax policy (carbon tax) and carbon emission trading policy (cap-and-trade) (Shi et al., 2019). Among them, the carbon cap-and-trade policy allows enterprises to trade carbon emission rights freely, which makes carbon emission a kind of "soft restriction", and becomes one of the most common carbon emission reduction policies implemented worldwide and the policy with the most obvious emission reduction effect (Kuiti et al., 2020; Kushwaha et al., 2020).

2. Literature review

At the level of enterprise decision, scholars, such as Schultz and Williamson (2005) study the impact of climate change and various carbon emission reduction policies on enterprise decision. They proved that enterprises must consider carbon emission rights as resources as important as capital, human resources, products and services from the two aspects of direct and potential impact, and puts forward that enterprises should attach great importance to the impact of carbon emission reduction policies. Based on the Newsboy Model, Zhang and Xu (2013) established a new profit maximization model to address stochastic demand and carbon emission reduction constraints and the multi-production decision problem of manufacturing enterprises under carbon trading policy constraints. Toptal et al. (2014) studied the joint decision of inventory replenishment and green technology investment by retail enterprises under three major carbon emission reduction policies. Du et al. (2015) studied the coordination of a two-echelon supply chain consisting of a supplier with carbon emission rights and an emission-dependent manufacturing enterprise under the constraints of a carbon trading policy and proved that carbon emission rights will be the key factor for the sustainable production of emission-dependent manufacturing enterprises by using the supply chain coordination mechanism designed by Non-cooperative Game theory. Ma et al. (2017) extended the newsboy model and incorporated green technology input into the model to study the intertemporal production decision of manufacturing enterprises with two production cycles under the constraint of a carbon trading policy. Olatunji et al. (2019) took some automobile manufacturing company in the United Kingdom as the case study, which reveals how to maintain the competitive advantage of carbon efficiency in the supply chain.

The research also revealed the necessity of implementing carbon reduction strategies in business development, and determined the influencing factors and obstacles related to the automobile manufacturing in the supply chain through case analysis and comparison of different companies. Research on the impact of major carbon emission reduction policies on corporate decision under different constraints is rich. However, most of the existing studies have focused only on the decision of a single enterprise or the joint decision of upstream and downstream enterprises in the supply chain.

The reality is that there are many markets where two or more enterprises produce products with the same functions, such as traditional energy, electricity, petroleum, the chemical industry and others. Enterprises in these markets should consider not only the regulation of carbon emission reduction policies but also the competition of their peers in the market. Therefore, this paper takes the carbon emission-dependent enterprises in this kind of market as the research object, and studies how to make price and quantity decisions on the premise of realizing the carbon emission reduction target under the three main carbon emission reduction policy constraints of carbon cap, carbon cap-and-trade, and carbon tax.

3. Problem description and hypothesis

On one hand, traditional energy markets, such as electricity, petroleum, chemical etc. have the typical characteristics of an **oligopoly**, such as Royal Dutch Shell and America's Exxon Mobil in energy industry, China National Petroleum Group Co., LTD., China Petroleum and Chemical Corporation and BP in oil industry, China State Grid Corporation and China Southern Power Grid Co., LTD., in power industry, America`s Dow, Dupont and Germany`s BASF in chemical industry. On the other hand, the traditional energy market has characteristics of carbon dioxide emission dependence due to the dependence on traditional coal, oil and other raw materials. This paper conducts research under the consideration of a carbon dioxide emission-dependent market represented by the traditional energy, power, petroleum and chemical industries. The products on this market are supplied by two oligarchs, and the market competition is an oligopoly. The two oligarchs have their own distribution channels, and the prices of their products are competitive in the market.

Each produces a single product with incomplete homogeneity, and the product difference comes from the different carbon dioxide emissions per unit. During the production process, they are subject to the maximum carbon emission quota stipulated by the government, that is, the carbon limit K_i (>0), in which i = 1, 2. In order to express conveniently, the meanings of the symbols in the model are shown in Table 1. The above parameters must meet certain conditions to ensure the practical significance of the model. Therefore, it is assumed that:

(1) The product carbon emissions per unit of oligopoly i is common knowledge, i = 1, 2 and $e_1<e_2$;

(2) K_i/e_1 is defined as the maximum production of oligopoly i, $Q_0 \geq \frac{K_i}{e_i}+\frac{K_j}{e_j}$, $i=\{1,2\}$, and j = 3-i;

(3) Assuming that oligarchs must maintain normal production and are rational, they will weigh the benefits and costs brought by carbon emission trading.

4. Basic model

4.1. Description of basic model

A decision model in a duopoly competitive market exists when the production process is not constrained by carbon emission reduction policies (Choi, 1991). Demand is a linear function of price, so demand increases with the decrease in market price and decreases with the decrease of a competitor's price.

Table 1. Symbols and meanings

Parameter	***Parametric Meaning***	***Parameter***	***Parametric Meaning***
Q_0	Market Capacity	c_1	Oligopoly 1 Unit cost of production
d_1	Oligopoly 1 Market Deman of the Products	c_2	Oligopoly 2 Unit cost of production
d_2	Oligopoly 2 Market Demand of the Products	K_1	Oligopoly 1 The Government's Maximum Carbon Emission Quota
p_1	Oligopoly 1 Market Price of the Products	K_2	Oligopoly 2 The Government's Maximum Carbon Emission Quota
p_2	Oligopoly 2 Market Price of the Products	w	Transaction Price of Carbon Emissions Per Unit
e_1	Oligopoly 1 Unit Carbon Emissions of the Products	π_1	Oligopoly 1 Profits
e_2	Oligopoly 2 Unit Carbon Emissions of the Products	π_2	Oligopoly 2 Profits
t_s	Taxes Imposed by the Government on Carbon Emissions Per Unit		

β is the price competition coefficient, and $0 \le \beta \le 1$. The larger β becomes, the more intense the price competition between the two products will be. In this case, the demand function of the two oligarchs is as Eq. (1):

$$d_i^{NC}\left(p_i, p_j\right) = Q_0 - p_i + \beta\left(p_j - p_i\right);\ i = \{1,2\},\ j = 3 - i \tag{1}$$

The profit functions of the two oligarchs are Eq. (2):

$$\pi_i^{NC} = \left(Q_0 - p_i + \beta\left(p_j - p_i\right)\right)\left(p_i - c_i\right);\ i = \{1,2\},\ j = 3 - i \tag{2}$$

In this case, the goal of decision is to maximize profits. The first-order optimal condition of Eq. (2) is Eq. (3):

$$\begin{cases} Q_0 + p_2\beta + c_1(1+\beta) - 2p_1(1+\beta) = 0 \\ Q_0 + p_1\beta + c_2(1+\beta) - 2p_2(1+\beta) = 0 \end{cases} \tag{3}$$

The optimal pricing and quantity decision of the two oligarchs can be obtained by solving Eq. (4):

$$\begin{cases} p_1^{*NC} = \dfrac{c_2\beta(1+\beta) + 2c_1(1+\beta)^2 + Q_0(2+3\beta)}{4+8\beta+3\beta^2} \\ p_2^{*NC} = \dfrac{c_1\beta(1+\beta) + 2c_2(1+\beta)^2 + Q_0(2+3\beta)}{4+8\beta+3\beta^2} \\ q_1^{*NC} = \dfrac{(1+\beta)\left(c_2\beta(1+\beta) + Q_0(2+3\beta) - c_1\left(2+4\beta+\beta^2\right)\right)}{(2+\beta)(2+3\beta)} \\ q_2^{*NC} = \dfrac{(1+\beta)\left(c_1\beta(1+\beta) + Q_0(2+3\beta) - c_2\left(2+4\beta+\beta^2\right)\right)}{(2+\beta)(2+3\beta)} \\ \pi_1^{*NC} = \dfrac{(1+\beta)\left(c_2\beta(1+\beta) + Q_0(2+3\beta) - c_1\left(2+4\beta+\beta^2\right)\right)^2}{(2+\beta)^2(2+3\beta)^2} \\ \pi_2^{*NC} = \dfrac{(1+\beta)\left(c_1\beta(1+\beta) + Q_0(2+3\beta) - c_2\left(2+4\beta+\beta^2\right)\right)^2}{(2+\beta)^2(2+3\beta)^2} \end{cases} \tag{4}$$

Therefore, in a duopoly competitive market, when the duopoly is not constrained by carbon emission reduction policies, there is an optimal price to satisfy the market, and there is also an optimal quantity and profit determined by the optimal price.

4.2. Carbon Cap policy model

Carbon cap policy means that in a competitive market, the two oligarchs are subject to the maximum carbon emission quota stipulated by the government, that is, carbon cap K_i (>0), in which *i=1, 2*. When the respective carbon emission quotas of the two oligarchs are insufficient or excessive, they cannot buy or sell carbon emission rights.

At this point, the profit function of oligopoly 1 is as Eq. (5):

$$\begin{cases} \pi_1^{CX} = d_1\left(p_1, p_2\right)\left(p_1 - c_1\right) \\ s.t.\, e_1 d_1 \le K_1 \end{cases} \tag{5}$$

The profit function of oligopoly 2 is as Eq. (6):

$$\begin{cases} \pi_2^{CX} = d_2\left(p_2, p_1\right)\left(p_2 - c_2\right) \\ s.t.\, e_2 d_2 \le K_2 \end{cases} \tag{6}$$

In this case, the decision goals of oligopoly 1 and 2 are to maximize profits.

Proposition 1: In the duopoly competitive market with carbon quota policy constraints, when the carbon quota is K_i=1 (i = 1, 2), the optimal output-pricing decision of the duopoly exists and is unique.

Proof:

Lagrange function of oligarch i is $L_i\left(p_i, p_j, \gamma_i^{CX}\right)$ in which γ_i^{CX} is the Lagrange coefficient $\gamma_i^{CX} > 0$, i = 1, 2, and j = 3 - i.

$$L_i\left(p_i, p_j, \gamma_i^{CX}\right) = \left(Q_0 - p_i + \beta\left(p_j - p_i\right)\right)\left(p_i - c\right) + \gamma_i^{CX}\left[e_i\left(Q_0 + p_i - \beta\left(p_j - p_i\right)\right) - K_i\right]$$

By solving the optimal conditions, the Lagrange coefficients are (Eq. 7):

$$\begin{cases} \gamma_1^{CX} = \dfrac{K_1 + c_1 e_1(1+\beta)}{(1+\beta)e_1^2} + \dfrac{e_1 K_2\beta + e_2\left(K_1(1+\beta) - e_1 Q_0(1+2\beta)\right)}{e_1^2\left(e_2 + 2e_2\beta\right)} \\ \gamma_2^{CX} = \dfrac{e_2 K_1\beta(1+\beta) + c_2 e_1 e_2\left(1+3\beta+2\beta^2\right) + e_1\left(K_2\left(2+4\beta+\beta^2\right) - e_2 Q_0\left(1+3\beta+2\beta^2\right)\right)}{e_1 e_2^2(1+\beta)(1+2\beta)} \end{cases} \tag{7}$$

The first derivative conditions (Eq. 8) are obtained using simultaneous Eqs. (5-6):

$$\begin{cases} -2p_1 + Q_0 - 2p_1\beta + p_2\beta + c_1(1+\beta) - e_1\gamma_1^{CX}(1+\beta) = 0 \\ -K_1 + e_1\left(Q_0 + p_2\beta - p_1(1+\beta)\right) = 0 \\ -K_2 + e_2\left(Q_0 + p_1\beta - p_2(1+\beta)\right) = 0 \\ -2p_2 + Q_0 + p_1\beta - 2p_2\beta + c_2(1+\beta) - e_2\gamma_2^{CX}(1+\beta) = 0 \end{cases} \quad (8)$$

By solving the equation, the optimal pricing and quantity decisions of the two oligarchs are as Eq. (9):

$$\begin{cases} p_1^{*cx} = Q_0 - \frac{1+\beta}{1+2\beta}\frac{K_1}{e_1} - \frac{\beta}{1+2\beta}\frac{K_2}{e_2} \\ p_2^{*cx} = Q_0 - \frac{1+\beta}{1+2\beta}\frac{K_2}{e_2} - \frac{\beta}{1+2\beta}\frac{K_1}{e_1} \\ q_1^{*cx} = \frac{K_1}{e_1} \\ q_2^{*cx} = \frac{K_2}{e_2} \\ \pi_1^{*cx} = -\frac{K_1\left(e_1K_2\beta + c_1e_1e_2(1+2\beta) + e_2\left(K_1 - e_1Q_0 + K_1\beta - 2e_1Q_0\beta\right)\right)}{e_1^2\left(e_2 + 2e_2\beta\right)} \\ \pi_2^{*cx} = -\frac{K_2\left(e_2K_2\beta + c_2e_1e_2(1+2\beta) + e_1\left(K_2 - e_2Q_0 + K_2\beta - 2e_2Q_0\beta\right)\right)}{e_2^2\left(e_1 + 2e_1\beta\right)} \end{cases} \quad (9)$$

Therefore, in the duopoly competitive market under the restriction of a carbon quota policy, when the government formulates the carbon quota as K_i (i =1, 2), the optimal production-pricing decision of the duopoly exists and is unique.

Evidence obtained

Corollary 1: In the duopoly competition market under the restriction of a carbon cap policy when $K_1/e_1 \geq K_2/e_2$, $p_1^{*CX} \leq p_2^{*CX}$; otherwise, $p_1^{*CX} > p_2^{*CX}$ and $\left(p_1^{*CX} - p_2^{*CX}\right)$ is inversely proportional to β.

Proof:

Becaus $\begin{cases} p_1^{*CX} = Q_0 - \frac{1}{1+2\beta}\left(\frac{K_1}{e_1}\right) - \frac{\beta}{1+2\beta}\left(\frac{K_1}{e_1} + \frac{K_2}{e_2}\right) \\ p_2^{*CX} = Q_0 - \frac{1}{1+2\beta}\left(\frac{K_2}{e_2}\right) - \frac{\beta}{1+2\beta}\left(\frac{K_1}{e_1} + \frac{K_2}{e_2}\right) \end{cases}$,

then $p_1^{*CX} - p_2^{*CX} = \frac{K_2/e_2 - K_1/e_1}{1+2\beta}$,

therefore, when $K_1/e_1 \geq K_2/e_2$, $p_1^{*CX} \leq p_2^{*CX}$; when $K_1/e_1 < K_2/e_2$, $p_1^{*CX} > p_2^{*CX}$.

Therefore, in the duopoly competition market under the restriction of a carbon cap policy, when $K_1/e_1 \geq K_2/e_2$, $p_1^{*CX} \leq p_2^{*CX}$; otherwise, $p_1^{*CX} > p_2^{*CX}$, and $\left(p_1^{*CX} - p_2^{*CX}\right)$ is inversely proportional to β.

Evidence obtained

The above propositions and inferences show that the optimal market price of products is affected by production capacity, product carbon emission per unit and competition coefficient under a carbon quota constraint. When the carbon quota is fixed, the smaller the product carbon emission per unit, the higher the capacity under the carbon quota constraint, and the lower the market price of the product. When the production capacity is fixed, the more intense, the competition between the two oligarchs, the smaller the market price difference of the products.

4.3. Carbon Cap-and-Trade policy model

A carbon cap-and-trade policy means that in a competitive market, the two oligarchs are subject to the maximum carbon emission quota stipulated by the government, that is, carbon cap K_i (>0), in which i=1, 2. Unlike the carbon cap policy, when the oligarchs produce the products, those oligarchs with insufficient carbon quotas can purchase carbon emission rights to further organize production, while those oligarchs with sufficient carbon quotas can sell surplus carbon quotas for profit. This paper assumes that oligarchs cannot completely abandon production. Therefore, enterprises can conduct carbon emission rights trading respectively through external trading and internal trading. First, the increase in oligopoly i from the definition of unit carbon emission rights is $\theta_i\left(p_i, p_{3-i}\right) = \frac{\partial \pi_i}{e_i \partial p_i}; i = \{1,2\}$, and w is the trading price of unit carbon emission rights; therefore:

- When $\theta_i\left(p_i, p_{3-i}\right) < w$, under the constraints of a carbon cap-and-trade policy, the profit increase of acquiring one more unit of carbon emission rights is less than the cost of purchasing carbon emission rights, and oligopoly i will consider selling carbon emission rights for profit.

- When $\theta_i\left(p_i, p_{3-i}\right) > w$, under the constraints of a carbon cap-and-trade policy, the profit increase of acquiring one more unit of carbon emission rights is greater than the cost of purchasing carbon emission rights, and oligopoly i will consider purchasing carbon emission rights to produce more products.

- When $\theta_i\left(p_i, p_{3-i}\right) = w$, under the constraints of a carbon cap-and-trade policy, the increase in profits from acquiring one more unit of carbon emission rights equals the cost of purchasing carbon emission rights, and oligopoly i will not consider trading carbon emission rights.

- Furthermore, when the two oligarchs conduct price and output competition:

$\theta_i\left(p_i, p_{3-i}\right) > \theta_{3-i}\left(p_{3-i}, p_i\right)$. This means that under the restriction of a carbon cap-and-trade policy, the profit increase of the oligopoly i from acquiring one more unit of carbon emission rights is less than the cost of purchasing carbon emission rights, and they will consider selling carbon emission rights. Another oligarch will consider purchasing a carbon emission right if it gains more profits than the cost of purchasing a carbon emission right. On the other hand, this is also true. At this point, there is a trade between the two oligarchs in the price competition of carbon emission rights, which is called the internal trade of carbon emission rights in this paper. Otherwise, it is considered as the external trade of carbon emission rights.

4.3.1. External trade

Both oligarchs are constrained by carbon cap-and-trade policies. They can trade carbon emission rights through the external carbon emission trading market. The price of carbon emission trading is determined by the market. The unit price of carbon emission trading is w^{CW}, which is the same for the two oligarchs, and the trading quantity is e_i^{CW}, $i=1,2$ (Eq. 10).

$$\begin{cases} \pi_1^{CW} = d_1(p_1,p_2)(p_1-c_1)-w^{CW}e_1^{CW} \\ s.t.\ e_1 d_1(p_1,p_2) = K_1+e_1^{CW} \end{cases} \tag{10}$$

The profit function of oligopoly 2 is as Eq. (11):

$$\begin{cases} \pi_2^{CW} = d_2(p_2,p_1)(p_2-c_2)-w^{CW}e_2^{CW} \\ s.t. e_2 d_2(p_2,p_1) = K_2+e_2^{CW} \end{cases} \tag{11}$$

In this case, the goal of decision for both oligopolies 1 and 2 is to maximize profits.

Proposition 2: In the case of external trading, in the duopoly competitive market with carbon quota policy constraints, when the carbon quota is K_i $(i=1,2)$ the optimal pricing and quantity decision of the duopoly exists and is unique.

Proof:

The Lagrange function is $L_i(p_i,p_j,\gamma_i^{CW})$, γ_i^{CW}, in which γ_i^{CX} is the Lagrange coefficient, $\gamma_i^{CW}>0$, *i = 1, 2*, and *j = 3-i*.

$$L_i(p_i,p_j,\gamma_i^{CW}) = \\ \left(Q_0-p_i+\beta(p_j-p_i)\right)(p_i-c_i)+\gamma_i^{CW}\left[e_i\left(Q_0+p_i-\beta(p_j-p_i)\right)-K_i-e_i^{CW}\right]-w^{CW}e_i^{CW}$$

By solving the optimal conditions, the Lagrange coefficients are Eq. (12):

$$\begin{cases} \gamma_1^{CW} = \dfrac{\begin{array}{c} e_1\left(K_2+e_2^{CW}\right)\beta(1+\beta)+c_1e_1e_2\left(1+3\beta+2\beta^2\right)+ \\ e_2\left(K_1\left(2+4\beta+\beta^2\right)+e_1^{CW}\left(2+4\beta+\beta^2\right)-e_1Q_0\left(1+3\beta+2\beta^2\right)\right) \end{array}}{e_1^2e_2(1+\beta)(1+2\beta)} \\ \gamma_2^{CW} = \dfrac{\begin{array}{c} e_2\left(K_1+e_1^{CW}\right)\beta(1+\beta)+c_2e_1e_2\left(1+3\beta+2\beta^2\right)+ \\ e_1\left(K_2\left(2+4\beta+\beta^2\right)+e_2^{CW}\left(2+4\beta+\beta^2\right)-e_2Q_0\left(1+3\beta+2\beta^2\right)\right) \end{array}}{e_1e_2^2(1+\beta)(1+2\beta)} \end{cases} \tag{12}$$

The first derivative conditions (Eq. 13) are obtained using simultaneous Eqs. (10-11):

$$\begin{cases} -2p_1+Q_0-2p_1\beta+p_2\beta+c_1(1+\beta)-e_1\gamma_1^{CX}(1+\beta)=0 \\ -e_1^{CW}-K_1+e_1\left(Q_0+p_2\beta-p_1(1+\beta)\right)=0 \\ -e_2^{CW}-K_2+e_2\left(Q_0+p_1\beta-p_2(1+\beta)\right)=0 \\ -2p_2+Q_0+p_1\beta-2p_2\beta+c_2(1+\beta)-e_2\gamma_2^{CX}(1+\beta)=0 \end{cases} \tag{13}$$

By solving the equation, the optimal pricing and quantity decisions of the two oligarchs are as Eq. (14):

$$\begin{cases} p_1^{*CW} = Q_0-\dfrac{1+\beta}{1+2\beta}\dfrac{\left(K_1+e_1^{CW}\right)}{e_1}-\dfrac{\beta}{1+2\beta}\dfrac{\left(K_2+e_2^{CW}\right)}{e_2} \\ p_2^{*CW} = Q_0-\dfrac{1+\beta}{1+2\beta}\dfrac{\left(K_2+e_2^{CW}\right)}{e_2}-\dfrac{\beta}{1+2\beta}\dfrac{\left(K_1+e_1^{CW}\right)}{e_1} \\ q_1^{*CW} = \dfrac{e_1^{CW}+K_1}{e_1} \\ q_2^{*CW} = \dfrac{e_2^{CW}+K_2}{e_2} \\ \pi_1^{*CW} = -\dfrac{\begin{array}{c} e_1\left(K_1+e_1^{CW}\right)\left(K_2+e_2^{CW}\right)\beta+c_1e_1e_2\left(K_1+e_1^{CW}\right)(1+2\beta)+ \\ e_2\begin{pmatrix} K_1^2(1+\beta)+K_1\left(2e_1^{CW}(1+\beta)-e_1Q_0(1+2\beta)\right)+ \\ e_1^{CW}\left(e_1^{CW}(1+\beta)+e_1\left(-Q_0+e_1w^{CW}\right)(1+2\beta)\right) \end{pmatrix} \end{array}}{e_1^2\left(e_2+2e_2\beta\right)} \\ \pi_2^{*CW} = -\dfrac{\begin{array}{c} e_2\left(K_1+e_1^{CW}\right)\left(K_2+e_2^{CW}\right)\beta+c_2e_1e_2\left(K_2+e_2^{CW}\right)(1+2\beta)+ \\ e_1\begin{pmatrix} k_2^2(1+\beta)+k_2\left(2e_2^{CW}(1+\beta)-e_2Q_0(1+2\beta)\right)+ \\ e_2^{CW}\left(e_2^{CW}(1+\beta)+e_2\left(-Q_0+e_2w^{CW}\right)(1+2\beta)\right) \end{pmatrix} \end{array}}{e_2^2\left(e_1+2e_1\beta\right)} \end{cases} \tag{14}$$

Therefore, in the case of external trading and considering the duopoly competitive market under the restriction of a carbon cap-and-trade policy, when the government formulates the carbon cap as K_i (i = 1, 2), the optimal pricing and quantity decision of the duopoly exists and is unique.

Evidence obtained

Corollary 2: The profit function of the oligopoly is a concave function e_i^{CW}, then the optimal carbon emission trading quantity of oligopoly i is Eq. (15):

$$e_i^{*CW} = \frac{-2c_ie_i\left(1+3\beta+2\beta^2\right)+e_i(1+2\beta)\left(\left(c_j+e_jw^{CW}\right)\beta-2e_iw^{CW}(1+\beta)+Q_0(2+\beta)\right)}{4+8\beta+3\beta^2}-K_i$$

$i = 1, 2;\ j = 3 - i.$ (15)

Proof:

The first derivative of the profit function after the trading of carbon emission rights by oligopoly i is Eq. (16):

$$\frac{\partial\pi_i}{\partial e_i^{CW}} = -\frac{\begin{array}{c} e_i\left(K_j+e_j^{CW}\right)\beta+c_ie_ie_j(1+2\beta)+ \\ e_j\left(2K_i(1+\beta)+2e_i^{CW}(1+\beta)+e_i\left(-Q_0+e_iw^{CW}\right)(1+2\beta)\right) \end{array}}{e_i^2\left(e_j+2e_j\beta\right)} \tag{16}$$

The second derivative is Eq. (17):

$$\frac{\partial^2\pi_i}{\partial e_i^{2CW}} = -\frac{2(1+\beta)}{\left(e_i^{CW}\right)^2(1+2\beta)} < 0 \tag{17}$$

According to the maximum theorem, we can know that the profit of oligopoly i has a maximum value. When:

$$\frac{\partial \pi_i}{\partial e_i^{CW}}=0,$$

$$e_i^{*CW}=\frac{-2c_ie_i\left(1+3\beta+2\beta^2\right)+e_i\left(1+2\beta\right)\left(\left(c_j+e_jw^{CW}\right)\beta-2e_iw^{CW}\left(1+\beta\right)+Q_0\left(2+\beta\right)\right)}{4+8\beta+3\beta^2}-K_i$$

Therefore, the profit function of the oligopoly is a concave function e_i^{CW}, then the optimal carbon emission trading quantity of the oligopoly i is Eq. (18):

$$e_i^{*CW}=\frac{-2c_ie_i\left(1+3\beta+2\beta^2\right)+e_i\left(1+2\beta\right)\left(\left(c_j+e_jw^{CW}\right)\beta-2e_iw^{CW}\left(1+\beta\right)+Q_0\left(2+\beta\right)\right)}{4+8\beta+3\beta^2}-K_i$$

$i=\{1,2\}$, $j=3-i$. (18)

Evidence obtained

Corollary 3: In the case of external trading and considering the duopoly competitive market under the restriction of a carbon cap-and-trade policy, when $\frac{K_2+e_2^{CW}}{e_2}\geq\frac{K_1+e_1^{CW}}{e_1}$, $p_1^{*CW}\geq p_2^{*CW}$, otherwise $p_1^{*CW}<p_2^{*CW}$, and $\left(p_1^{*CW}-p_2^{*CW}\right)$ is inversely proportional to β.

Proof:

Because:

$$\begin{cases} p_1^{*CW}=Q_0-\frac{1+\beta}{1+2\beta}\frac{\left(K_1+e_1^{CW}\right)}{e_1}-\frac{\beta}{1+2\beta}\frac{\left(K_2+e_2^{CW}\right)}{e_2} \\ p_2^{*CW}=Q_0-\frac{1+\beta}{1+2\beta}\frac{\left(K_2+e_2^{CW}\right)}{e_2}-\frac{\beta}{1+2\beta}\frac{\left(K_1+e_1^{CW}\right)}{e_1} \end{cases},$$

then, $p_1^{*CW}-p_2^{*CW}=\frac{\frac{K_2+e_2^{CW}}{e_2}-\frac{K_1+e_1^{CW}}{e_1}}{1+2\beta}$;

therefore, when $\frac{K_2+e_2^{CW}}{e_2}\geq\frac{K_1+e_1^{CW}}{e_1}$, $p_1^{CW}\geq p_2^{CW}$; and when $\frac{K_2+e_2^{CW}}{e_2}<\frac{K_1+e_1^{CW}}{e_1}$, $p_1^{CW}<p_2^{CW}$.

Therefore, in the case of external trading and considering the duopoly competitive market under the restriction of a carbon quota policy, when $\frac{K_2+e_2^{CW}}{e_2}\geq\frac{K_1+e_1^{CW}}{e_1}$, $p_1^{*CW}\geq p_2^{*CW}$, otherwise $p_1^{*CW}<p_2^{*CW}$, and $\left(p_1^{*CW}-p_2^{*CW}\right)$ is inversely proportional to β.

Evidence obtained

The above propositions and inferences show that, in the case of external trading, the optimal market price of products is affected by the capacity after external trading of carbon emission rights, product carbon emission per unit and competition coefficient. The smaller the product carbon emission per unit is, the higher the capacity after external trading of carbon emission rights and the lower the market price of products. When the capacity of carbon emission rights is fixed after external trading, the more intense the competition between the two oligarchs is, the smaller the price difference in the product market. At the same time, oligarchs are willing to buy additional carbon emission rights in the external carbon emission trading market to organize production and competition as long as the profit of unit carbon emission rights is higher than the external carbon emission trading price. Moreover, the more competitive the two oligarchs are, the more willing they will be to buy additional carbon emission rights in the external carbon emission trading market.

4.3.2. Internal trade

The oligarchs are constrained by carbon cap-and-trade policies, and they can consider cooperating in carbon trading within the market. When selling or purchasing carbon emission rights, they will obtain the trading quantity and price of carbon emission rights according to their own profit maximization decision, but they will also be constrained by the market price of the other party. The unit carbon emission trading price is w^{CN}, trading quantity is e^{CN}. Suppose oligopoly 1 produces low-carbon products, without loss of generality, we assume that oligopoly 1 purchases carbon emission rights from oligopoly 2. At this point, the profit function of oligopoly 1 is as given by Eq. (19):

$$\begin{cases} \pi_1^{CN}=d_1\left(p_1,p_2\right)\left(p_1-c_1\right)-w^{CN}e^{CN} \\ s.t.\ \ e_1d_1\left(p_1,p_2\right)=K_1+e^{CN} \end{cases} \tag{19}$$

The profit function of oligopoly 2 is as given by Eq. (20):

$$\begin{cases} \pi_2^{CN}=d_2\left(p_2,p_1\right)\left(p_2-c_2\right)+w^{CN}e^{CN} \\ s.t.\ \ e_2d_2\left(p_2,p_1\right)=K_2-e^{CN} \end{cases} \tag{20}$$

In this case, the decision goals for both oligopoly 1 and 2 are to maximize profits.

Proposition 3: In the case of internal trading, in the duopoly competitive market with carbon cap-and-trade policy constraints, when the carbon cap is $K_i\ (i=1,2)$, the optimal output-pricing decision of the duopoly exists and is unique.

Proof:

Lagrange function is $L_i\left(p_i,p_j,\gamma_i^{CN}\right)$, in which γ_i^{CN} is the Lagrange coefficient,, $\gamma_i^{CN}>0$, $i,j\in\{1,2\}$; $j=3-i$。

$$\begin{cases} L_i\left(p_i,p_j,\gamma_i^{CN}\right)= \\ \quad\left(Q_0-p_i+\beta\left(p_j-p_i\right)\right)\left(p_i-c_i\right)+\gamma_i^{CN}\left[e_i\left(Q_0-p_i+\beta\left(p_j-p_i\right)\right)-K_i-e^{CN}\right]-w^{CN}e^{CN} \\ L_j\left(p_i,p_j,\gamma_i^{CN}\right)= \\ \quad\left(Q_0-p_j+\beta\left(p_i-p_j\right)\right)\left(p_j-c_j\right)+\gamma_j^{CN}\left[e_i\left(Q_0-p_i+\beta\left(p_i-p_j\right)\right)-K_j+e^{CN}\right]+w^{CN}e^{CN} \end{cases}$$

By solving the optimal conditions, the Lagrange coefficients are Eq. (21):

$$\begin{cases} \gamma_1^{CN} = \dfrac{\begin{array}{c} e_1\left(K_2-e^{CN}\right)\beta(1+\beta)+c_1e_1e_2\left(1+3\beta+2\beta^2\right)+ \\ e_2\left(K_1\left(2+4\beta+\beta^2\right)+e^{CN}\left(2+4\beta+\beta^2\right)-e_1Q_0\left(1+3\beta+2\beta^2\right)\right)\end{array}}{e_1^2e_2(1+\beta)(1+2\beta)} \\ \gamma_2^{CN} = \dfrac{\begin{array}{c} e_2\left(K_1+e^{CN}\right)\beta(1+\beta)+c_2e_1e_2\left(1+3\beta+2\beta^2\right)- \\ e_1\left(-K_2\left(2+4\beta+\beta^2\right)+e^{CN}\left(2+4\beta+\beta^2\right)+e_2Q_0\left(1+3\beta+2\beta^2\right)\right)\end{array}}{e_1e_2^2(1+\beta)(1+2\beta)} \end{cases} \tag{21}$$

The first derivative conditions are obtained using simultaneous Eqs. (22):

$$\begin{cases} -2p_1+Q_0-2p_1\beta+p_2\beta+c_1(1+\beta)-e_1\gamma_1^{CX}(1+\beta)=0 \\ -e_1^{CN}-K_1+e_1\left(Q_0+p_2\beta-p_1(1+\beta)\right)=0 \\ -e_2^{CN}+K_2+e2\left(Q_0+p_1\beta-p_2(1+\beta)\right)=0 \\ -2p_2+Q_0+p_1\beta-2p_2\beta+c_2(1+\beta)-e_2\gamma_2^{CX}(1+\beta)=0 \end{cases} \tag{22}$$

By solving the equation, the optimal pricing and quantity decisions of the two oligarchs are as Eq. (23). Therefore, in the case of internal trading and considering the duopoly competitive market under the restriction of a carbon cap-and-trade policy, when the government formulates the carbon cap as K_i (i = 1, 2) the optimal production-pricing decision of the duopoly exists and is unique. In the case of internal trading and considering the duopoly competitive market under the restriction of a carbon cap-and-trade policy, when the government formulates the carbon cap as K_i (i = 1, 2) the optimal production-pricing decision of the duopoly exists and is unique.

$$\begin{cases} p_1^{*CN} = \dfrac{e_1\left(-K_2+e^{CN}\right)\beta-e_2\left(K_1+e^{CN}-e_1Q_0+K_1\beta+e^{CN}\beta-2e_1Q_0\beta\right)}{e_1e_2(1+2\beta)} \\ p_2^{*CN} = \dfrac{-e_2\left(K_1+e^{CN}\right)\beta+e_1\left(e^{CN}+e_2Q_0+e^{CN}\beta+2e_2Q_0\beta-K_2(1+\beta)\right)}{e_1e_2(1+2\beta)} \\ q_1^{*CN} = \dfrac{K_1+e^{CN}}{e_1} \\ q_2^{*CN} = \dfrac{K_2-e^{CN}}{e_2} \\ \pi_1^{*CN} = -\dfrac{\begin{array}{c} e_1\left(K_2-e^{CN}\right)\left(K_1+e^{CN}\right)\beta+c_1e_1e_2\left(K_1+e^{CN}\right)(1+2\beta)+ \\ e_2\begin{pmatrix} k_1^2(1+\beta)+k_1\left(2e^{CN}(1+\beta)-e_1Q_0(1+2\beta)\right)+ \\ e^{CN}\left(e^{CN}(1+\beta)+e_1\left(-Q_0+e_1w^{CN}\right)(1+2\beta)\right) \end{pmatrix}\end{array}}{e_1^2\left(e_2+2e_2\beta\right)} \\ \pi_2^{*CN} = -\dfrac{\begin{array}{c} e_2\left(K_2-e^{CN}\right)\left(K_1+e^{CN}\right)\beta+c_2e_1e_2\left(K_2-e^{CN}\right)(1+2\beta)+ \\ e_1\begin{pmatrix} k_2^2(1+\beta)-k_2\left(2e^{CN}(1+\beta)+e_2Q_0(1+2\beta)\right)+ \\ e^{CN}\left(e^{CN}(1+\beta)+e_2\left(Q_0-e_2w^{CN}\right)(1+2\beta)\right) \end{pmatrix}\end{array}}{e_2^2\left(e_1+2e_1\beta\right)} \end{cases} \tag{23}$$

Evidence obtained

Corollary 4: If the profit function of the oligopoly is a concave function e^{CN}, then $\dfrac{2e_i\beta-2e_j(1+\beta)}{e_i^2\left(e_j+2e_j\beta\right)}<0$, $i=\{1,2\}$, and $j=3-i$; when $e_1^{*CN}\le e_2^{*CN}$ the actual volume of trade is e_1^{*CN}; and when $e_1^{*CN}>e_2^{*CN}$ the actual volume of trade is e_2^{*CN}.

Proof:

The purchasing amount of the buyer of the carbon emission rights in the internal trading is e_1^{CN}, and the selling amount of the seller of the carbon emission rights in the internal trading is e_2^{CN}.

The first derivative of the profit function of oligopoly 1 after purchasing carbon emissions is Eq. (24):

$$\frac{\partial\pi_1}{\partial e_1^{CN}} = -\frac{\begin{array}{c} e_1\left(K_2-e_1^{CN}\right)\beta-e_1\left(K_1+e_1^{CN}\right)\beta+c_1e_1e_2(1+2\beta)+ \\ e_2\left(2K_1(1+\beta)+2e^{CN}(1+\beta)+e_1\left(-Q_0+e_1w^{CN}\right)(1+2\beta)\right)\end{array}}{e_1^2\left(e_2+2e_2\beta\right)} \tag{24}$$

Based on the previous assumption $e_1<e_2$, then

$$\frac{\partial^2\pi_1}{\partial e_1^{CN^2}} = \frac{2e_1\beta-2e_2(1+\beta)}{e_1^2\left(e_2+2e_2\beta\right)}<0,$$

so oligopoly 1 has the only optimal profit and must choose to purchase carbon emission rights.

When $\dfrac{\partial\pi_1}{\partial e_1^{CN}}=0$, the optimal purchase quantity is according Eq. (25):

$$e_1^{*CN} = -\frac{e_1\left(-K_1+K_2\right)\beta+c_1e_1e_2(1+2\beta)+e_2\left(2K_1(1+\beta)+e_1\left(-Q_0+e_1w^{CN}\right)(1+2\beta)\right)}{2\left(e_2-e_1\beta+e_2\beta\right)} \tag{25}$$

Similarly, the first derivative of the profit function of oligopoly 2 after trading carbon emission rights is Eq. (26):

$$\frac{\partial\pi_2}{\partial e_2^{CN}} = \frac{\begin{array}{c} e_2\left(K_1-K_2+2e_2^{CN}\right)\beta+c_2e_1e_2(1+2\beta)+ \\ e_1\left(2K_2(1+\beta)-2e_2^{CN}(1+\beta)+e_2\left(-Q_0+e_2w^{CN}\right)(1+2\beta)\right)\end{array}}{e_2^2\left(e_1+2e_1\beta\right)} \tag{26}$$

As can be seen from the previous assumption $e_1<e_2$:

$$\frac{\partial^2\pi_2}{\partial e_2^{CN^2}} = \frac{2e_2\beta-2e_1(1+\beta)}{e_2^2\left(e_1+2e_1\beta\right)}<0 \text{ or } \frac{\partial^2\pi_2}{\partial e_2^{CN^2}} = \frac{2e_2\beta-2e_1(1+\beta)}{e_2^2\left(e_1+2e_1\beta\right)}>0$$

For oligopoly 2,

when $-\dfrac{-2e_2\beta+2e_1(1+\beta)}{e_2^2\left(e_1+2e_1\beta\right)}<0$ and $\dfrac{\partial\pi_2}{\partial e_2^{CN}}=0$, the profit of oligopoly 2 reaches the maximum value. At this time, the optimal sales volume is given by Eq. (27):

$$e_2^{*CN} = \frac{e_2\left(K_1-K_2\right)\beta+c_2e_1e_2(1+2\beta)+e_1\left(2K_2(1+\beta)+e_2\left(-Q_0+e_2w^{CN}\right)(1+2\beta)\right)}{2\left(e_1+e_1\beta-e_2\beta\right)} \tag{27}$$

When $-\frac{-2e_2\beta+2e_1(1+\beta)}{e_2^2(e_1+2e_1\beta)}>0$ and $\frac{\partial\pi_2}{\partial e_2^{CN}}=0$, oligopoly 2 has a minimum profit and will not sell carbon emission rights.

Therefore, if the profit function of oligopoly is a concave function e_i^{CN}, then $\frac{2e_i\beta-2e_j(1+\beta)}{e_i^2(e_j+2e_j\beta)}<0$, $i=1$, 2, and $j=3-i$; when $e_i^{*CN}\le e_j^{*CN}$, the actual volume of trade is e_i^{*CN}; and when $e_i^{*CN}>e_j^{*CN}$, the actual volume of trade is e_j^{*CN}.

Evidence obtained

Corollary 5: In the case of internal trading, considering the duopoly competitive market under the restriction of a carbon quota policy when $\frac{K_2-e^{CN}}{e_2}\ge\frac{K_1+e^{CN}}{e_1}$, $p_1^{*CN}\ge p_2^{*CN}$, $\frac{K_2-e^{CN}}{e_2}<\frac{K_1+e^{CN}}{e_1}$, $p_1^{*CN}<p_2^{*CN}$ and $\left(p_1^{CN}-p_2^{CN}\right)$ is inversely proportional to β.

Proof:

Because:

$$\begin{cases} p_1^{CN}=\dfrac{e_1\left(-K_2+e^{CN}+e_2Q_0\right)\beta-e_2\left(K_1+e^{CN}-e_1Q_0\right)(1+\beta)}{e_1e_2(1+2\beta)} \\ p_2^{CN}=\dfrac{-e_2\left(K_1+e^{CN}\right)\beta+e_1\left(e^{CN}+e_2Q_0+e^{CN}\beta+2e_2Q_0\beta-K_2(1+\beta)\right)}{e_1e_2(1+2\beta)} \end{cases}$$

then, $p_1^{CN}-p_2^{CN}=\dfrac{\frac{K_2-e^{CN}}{e_2}-\frac{K_1+e^{CN}}{e_1}}{1+2\beta}$, therefore, when:

$\frac{K_2-e^{CN}}{e_2}\ge\frac{K_1+e^{CN}}{e_1}$, $p_1^{CN}\ge p_2^{CN}$;

and when $\frac{K_2-e^{CN}}{e_2}<\frac{K_1+e^{CN}}{e_1}$, $p_1^{CN}<p_2^{CN}$.

Therefore, in the case of internal trading, considering the duopoly competitive market under the restriction of carbon quota policy, when $\frac{K_2-e^{CN}}{e_2}\ge\frac{K_1+e^{CN}}{e_1}$, $p_1^{*CN}\ge p_2^{*CN}$; when $\frac{K_2-e^{CN}}{e_2}<\frac{K_1+e^{CN}}{e_1}$, $p_1^{*CN}<p_2^{*CN}$; and $p_i^{*CN}\,(i=1,2)$ is inversely proportional to β.

Evidence obtained

The above propositions and inferences show that, in the case of internal trading, the optimal market price of products is affected by the capacity after internal trading of carbon emission rights, the product unit carbon emission and the competition coefficient. As long as the price of the internal trading of unit carbon emission rights is within a certain range, that is, less than the profit of unit carbon emission rights of an oligopoly and higher than that of another oligopoly, the internal trading of carbon emission rights will occur. However, as the market competition intensifies, the revenue of oligarchs selling carbon emissions rights will decrease, and the sellers of carbon emissions rights will reduce their sales volume until the internal trading of carbon emissions rights ceases.

4.4. Carbon tax policy model

Carbon tax policy means that in a competitive market between two oligarchs, the government will tax the carbon dioxide emissions generated by the two oligarchs in the production process and achieve the goal of reducing carbon dioxide emissions through taxation. The unit carbon tax of carbon emissions is the same. Its profit function is as Eq. (28):

$$\pi_i^{CS}=\left(p_i-c_i-t_se_i\right)d_i\left(p_i,p_j\right);\ i=\{1,2\},\ j=3-i \tag{28}$$

In this case, the goal of decision for both oligarchs is to maximize profits.

Proposition 4: In the duopoly competitive market with carbon tax policy constraints, the optimal pricing and quantity decision of the duopoly exists and is unique.

Proof:

By substituting the market demand function of two oligarchs $d_i^{CS}\left(p_i,p_j\right)=Q_0-p_i+\beta\left(p_j-p_i\right)$, $i=\{1,2\}$, and $j=3-i$ into their profit function, we can obtain Eq. (29):

$$\begin{cases} \pi_1^{CS}=\left(p_1-c_1-t_se_1\right)\left(Q_0-p_1+\beta\left(p_2-p_1\right)\right) \\ \pi_2^{CS}=\left(p_2-c_2-t_se_2\right)\left(Q_0-p_2+\beta\left(p_1-p_2\right)\right) \end{cases} \tag{29}$$

The optimal pricing and quantity decision of the two oligarchs can be obtained by solving the equations as Eq. (30):

$$\begin{cases} p_1^{*CS}=\dfrac{2Q_0+c_2\beta+3Q_0\beta+c_2\beta^2+2c_1(1+\beta)^2+t_se_1\left(2+5\beta+3\beta^2\right)}{4+8\beta+3\beta^2} \\ p_2^{*CS}=\dfrac{2Q_0+c_1\beta+3Q_0\beta+c_1\beta^2+2c_2(1+\beta)^2+t_se_2\left(2+5\beta+3\beta^2\right)}{4+8\beta+3\beta^2} \\ \pi_1^{*CS}=\dfrac{(1+\beta)\left(2Q_0+c_2\beta+3Q_0\beta+c_2\beta^2-t_s(2+3\beta)-c_1\left(2+4\beta+\beta^2\right)\right)^2}{(2+\beta)^2(2+3\beta)^2} \\ \pi_2^{*CS}=\dfrac{(1+\beta)\left(2Q_0+c_1\beta+3Q_0\beta+c_1\beta^2-t_s(2+3\beta)-c_2\left(2+4\beta+\beta^2\right)\right)^2}{(2+\beta)^2(2+3\beta)^2} \\ q_1^{*CS}=\dfrac{(1+\beta)\left(2Q_0+c_2\beta+3Q_0\beta+c_2\beta^2-t_s(2+3\beta)-c_1\left(2+4\beta+\beta^2\right)\right)}{(2+\beta)(2+3\beta)} \\ q_2^{*CS}=\dfrac{(1+\beta)\left(2Q_0+c_1\beta+3Q_0\beta+c_1\beta^2-t_s(2+3\beta)-c_2\left(2+4\beta+\beta^2\right)\right)}{(2+\beta)(2+3\beta)} \end{cases} \tag{30}$$

Therefore, in the duopoly competitive market with carbon tax policy constraints, the optimal pricing and quantity decision of the duopoly exists and is unique.

Evidence obtained

Corollary 6: Considering the duopoly competition market under the restriction of a carbon tax policy, when $e_1<e_2$ and $c_1<c_2$, $p_1^{*CS}<p_2^{*CS}$, not vice versa.

Proof:

Because:

$$\begin{cases} p_1^{*CS} = \dfrac{2(1+\beta)(c_1+t_se_1+Q_0+c_1\beta+te_1\beta)+\beta(c_2+t_se_2+Q_0+c_2\beta+t_se_2\beta)}{4+8\beta+3\beta^2} \\ p_2^{*CS} = \dfrac{2Q_0+c_1\beta+t_se_1\beta+3Q_0\beta+c_1\beta^2+t_se_1\beta^2+2c_2(1+\beta)^2+2t_se_2(1+\beta)^2}{4+8\beta+3\beta^2} \end{cases}$$

then, $\left(p_1^{*CS}-p_2^{*CS}\right)=\dfrac{(2+3\beta)}{(2+\beta)}(e_1-e_2)t_s+\dfrac{(1+\beta)}{(2+3\beta)}(c_1-c_2)$;

when $e_1 < e_2$ and $c_1 < c_2$, $p_1^{*CS} < p_2^{*CS}$, not vice versa.

Therefore, considering the duopoly competition market under the restriction of a carbon tax policy, when $e_1 < e_2$ and $c_1 < c_2$, $p_1^{*CS} < p_2^{*CS}$, not vice versa.

Evidence obtained

The above propositions and inferences show that in the duopoly competition market under the restriction of a carbon tax policy, the essence of the carbon tax policy, a carbon emission reduction policy, is to increase the environmental cost on the basis of the original cost to enterprises. Therefore, the optimal market price difference between the two products is related to product cost, competition coefficient, product unit carbon emission and carbon tax rate. Usually, the production cost of products with low unit carbon emissions is high, so the price of the products is higher when the two oligarchs compete. Therefore, in the duopoly competition market under the restriction of a carbon tax policy, if the oligarchs wish to win the market and consumers through low prices, then they must use low carbon technology to effectively control and reduce production costs and product unit carbon emissions.

5. Numerical analysis

Considering the impact of carbon emission reduction policies on the decision of the two oligarchs, numerical examples can more intuitively show the market price, output and profit trend of the two oligarchy's products to provide a basis for the government to better formulate carbon emission reduction policies. The parameters are: Market capacity is Q_0 = 50 coefficient of competition is β [0,1]. The production cost and product unit carbon emission of oligopoly 1 are c_1=2 and e_1=3, respectively. The production cost and product unit carbon emission of oligopoly 2 are c_2=1 and e_2=4, respectively. The carbon quota and the unit carbon tax set by the government are K_1=K_2=20 and t_s=4 respectively.

The price of internal carbon emission trading and external carbon emission trading are $w_n = 12$ and $w_w = 5$ respectively. Based on these conditions, we can obtain the market price decision of the oligarchs under each carbon emission reduction policy in Fig. 1, the output decisions of the two oligarchs under the constraints of carbon emission reduction policies in Fig. 2, and the profit curves of the two oligarchs under carbon emission reduction policies in Fig. 3.

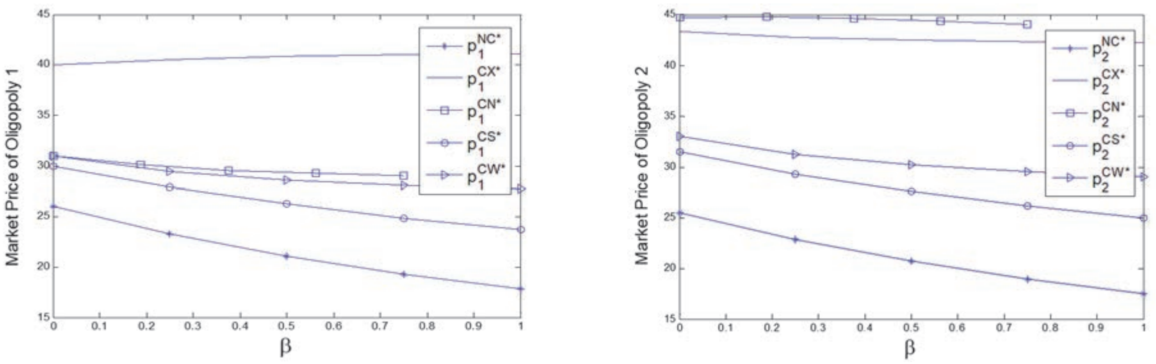
5.1. Basic model

The basic model is that in a duopoly competition market, the production process is not subject to carbon emission reduction policy constraints. In Figs. 1-3, the upper-right label *NC** shows the basic model case. According to formula (4), the price decision, output decision and profit curve of the two oligarchs under different competitive degrees β can be obtained.

Through numerical analysis, it can be found that, in this case, with the intensification of market competition, the market price of the two oligarchs will decline. When the competition coefficient is $\beta=1$, i.e., the most intense price competition, the market price of the two oligarchs is close. With the intensification of market competition, the output of the two oligarchs increases. When the competition coefficient is $\beta=1$, the output of the two oligarchs is close. With the intensification of market competition, the profits of both oligarchs decrease.

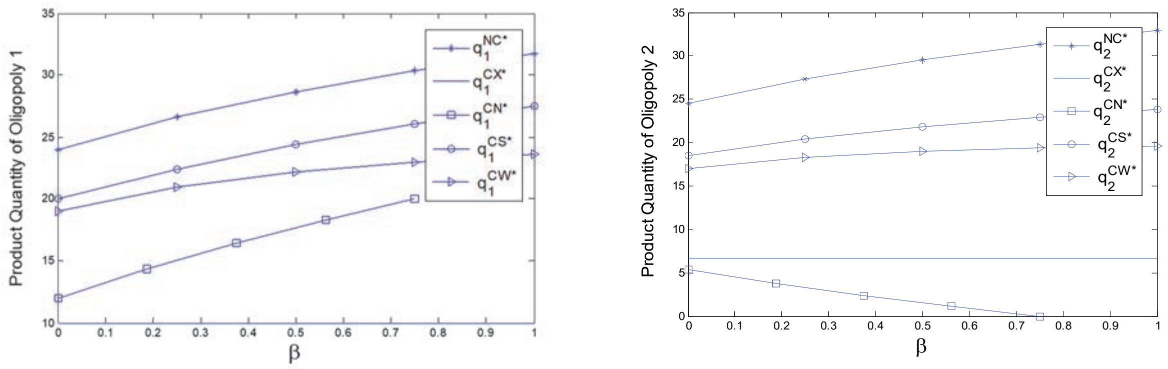
5.2. Carbon cap constraints

In the case of carbon cap policy constraints, in a competitive market, the production process of the two oligarchs is constrained respectively by the maximum carbon emission quotas set by the government. In Figs. 1-3, the upper-right label *CX** represents the carbon cap policy constraints.



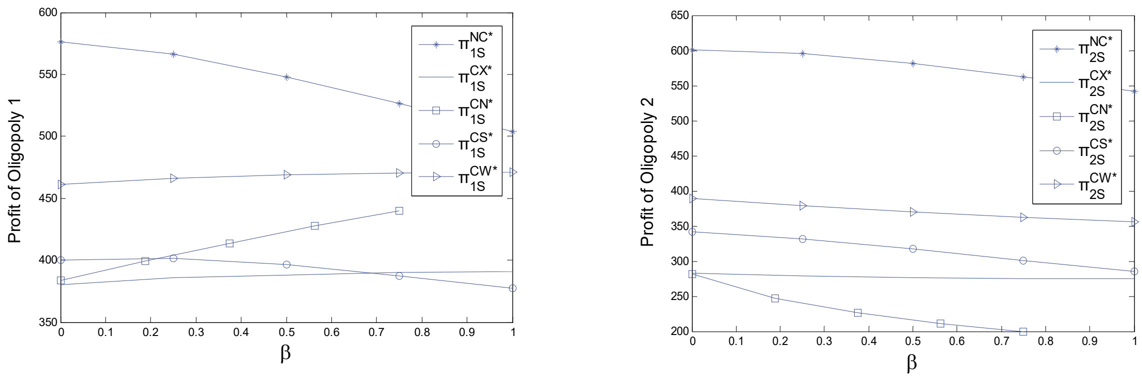
(a) (b)

Fig. 1. Market price decisions of two oligarchs: (a) represents market price decision of oligopoly 1, (b) represents market price decision of oligopoly 2



(a) (b)

Fig. 2. Output decisions of two oligarchs: (a) represents production decision of oligopoly 1, (b) represents production decision of oligopoly 2



(a) (b)

Fig. 3 Profit curves of two oligarchs: (a) represents profit curve of oligopoly 1, (b) represents profit curve of oligopoly 2

According to formula (7), we can obtain the market price decisions, output decisions and profit curves of the two oligarchs under different competition degrees β. Through numerical analysis, it can be found that in this case, with the intensification of market competition, the market price, output and profit of the two oligarchs show no significant change; that is, the carbon cap policy has no significant effect on the market price, output or profit of the two oligarchs.

5.3. Carbon Cap-and-Trade Policy Constraints

In the case of carbon cap-and-trade policy constraints, in a competitive market between two oligarchs, the production of the two oligarchs is constrained by the maximum carbon emission quotas set by the government, and oligarchs with insufficient carbon emission quotas can purchase carbon emission rights to further organize production, while oligarchs with sufficient carbon emission quotas can sell surplus carbon emission quotas for profit. In this paper, the trade is studied through two modes: internal trading and external trading. In Figs. 1-3, the upper-right labels *CN** and *CW** represent the internal trading and external trading of carbon emission rights, respectively. According to formula (10) and formula (13), the price decision, output decision and profit curve of the two oligarchs under different competitive degrees β can be obtained. Through numerical analysis, it can be found that in this case, with the intensification of market competition, when the competition coefficient β is large, the income of oligarchs selling carbon emission rights will decrease, and the seller of carbon emission rights will reduce the sale volume of carbon until the internal trading of carbon emission rights ceases. With the intensification of market competition, the market price of the two oligarchs shows no significant change in the case of internal trading. The market price of the two oligarchs declines more obviously in external trading than in internal trading. With the intensification of market competition, the outputs of both oligarchs increase due to external trading, in which the output of oligarch 2, which produces high carbon products, increases more slowly than that of oligarch 1, which produces low carbon products. With the intensification of market competition and in internal trading, the profit of oligopoly 1, which produces low carbon products, increases more obviously than that of oligopoly 2, which produces high carbon products, while the profit of oligopoly 2, which produces high carbon products, obviously decreases. In external trading, the profit of oligopoly 1, which produces low carbon products, increases slowly, while that of oligopoly 2, which produces high carbon products, decreases slowly.

5.4. Carbon Tax Policy constraints

Carbon tax constraint occurs when, in a market where two oligarchs compete, the government taxes the carbon dioxide emissions generated by the two oligarchs in the production process. In Figs. 1-3, the upper-right label CS^{*} indicates the carbon tax policy constraints. According to formula (15), the price decision, output decision and profit curve of the two oligarchs under different competitive degrees β can be obtained. Through numerical analysis, it can be found that in this case, with the intensification of market competition, the market prices and profits of the two oligarchs are declining, while the output is increasing.

5.5. Comparison of different carbon emission reduction policies

5.5.1. The impact of different carbon emission reduction policies on the market prices of two oligarchs

For oligopoly 1, which produces low carbon products, under the same competitive coefficient, the market price under the noncarbon emission reduction policy is lower than that under carbon emission reduction policies. Under the same market competition coefficient, among all carbon emission reduction policies, the market price under the restriction of a carbon tax policy is the lowest and the market price under the restriction of a carbon cap policy is the highest.

For oligopoly 2, which produces high carbon products, under the same competitive coefficient, the market price under the noncarbon emission reduction policy is lower than that under carbon emission reduction policies. Under the same market competition coefficient, among all carbon emission reduction policies, the market price under the restriction of a carbon tax policy is the lowest and the market price of internal trading under the restriction of a carbon cap-and-trade policy is the highest.

5.5.2. Impact of different carbon cap policies on the production quantity of the two oligarchs

For oligopoly 1, which produces low carbon products, under the same market competition coefficient, the output under a noncarbon emission reduction policy is higher than that under carbon emission reduction policies. Under the same market competition coefficient, among all carbon emission reduction policies, the output under the restriction of a carbon tax policy is the highest and the output under the carbon cap policy is the lowest.

For oligopoly 2, which produces high-carbon products, under the same market competition coefficient, the output without a carbon emission reduction policy is higher than that under carbon emission reduction policies. Under the same market competition coefficient, among all carbon emission reduction policies, the output under the restriction of a carbon tax policy is the highest and the output on the basis of internal trading under a carbon cap-and-trade policy is the lowest.

5.5.3 Impact of different carbon emission reduction policies on the profits of two oligarchs

For oligopoly 1, which produces low carbon products, regardless of whether the market competition coefficient is high or low, the profit under all carbon emission reduction policies is smaller than that under no carbon emission reduction policies. Under the constraints of all carbon emission reduction policies, the profit of external trading of carbon emission rights under the carbon cap-and-trade policy is the highest.

When the market competition coefficient is small, the profit under the carbon tax policy and the internal trading of carbon emission rights under the carbon cap-and-trade policy may be equal, that is, the profit impact of the two policies on oligopoly 1, which produces low carbon products, may be the same. When the market competition coefficient is large, the profit under the carbon cap policy may be equal to that under the carbon tax policy; that is, the two policies may have the same impact on the profit of oligopoly 1, which produces low carbon products.

For oligopoly 2, which produces high-carbon products, no matter if the market competition coefficient is high or low, the profit under all carbon emission reduction policies is smaller than that under no carbon emission reduction policies. Under the constraints of all carbon emission reduction policies, the profit of external trading of carbon emission rights under carbon cap-and-trade policy is the highest. When the market competition coefficient is small, the profit under the carbon cap-and-trade policy is close to that under the carbon tax policy. When the market competition coefficient is large, the profit under the carbon cap policy is close to that under the carbon tax policy.

6. Conclusions

Based on a carbon dioxide emission-dependent market represented by the traditional energy, electricity, petroleum and chemical industries, this paper studies the optimal price and competitive decision in the duopoly competition market under the constraints of three carbon emission reduction policies, carbon cap, carbon cap-and-trade, and carbon tax, and verifies the relevant conclusions through numerical analysis.

From the analysis, we can see that in the duopoly competition market constrained by three major carbon emission reduction policies, there are equilibrium solutions for both the price and output decisions of enterprises. Among them, the optimal decision of enterprises is affected by the capacity, product unit carbon emission and price competition coefficient under the restriction of the carbon cap policy. The optimal decision of enterprises under the restriction of a carbon cap-and-trade policy, whether external or internal, is affected by the capacity, product unit carbon emission and competition coefficient after carbon emission trading. The optimal market price difference of products under the restriction of a carbon tax policy is related to product cost, competition coefficient, product unit carbon emission and carbon tax rate.

Further research shows that because of the competition between the two oligarchic markets, enterprises require greater capacity to reduce the market price of products and gain a competitive advantage. For the government, when the carbon quota is set and market competition is not fierce, allowing the internal trading of carbon emission rights will generate the greatest profit for both suppliers and consumers and society as a whole. When market competition is fierce, the establishment of external trading of carbon emission rights will generate optimal benefits for the supply and demand sides and society as a whole.

From the perspective of energy saving and emission reduction, a carbon tax policy is more conducive to encouraging suppliers to use low carbon technology.

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