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ANALYSIS OF THE EVOLUTIONARY GAME OF DECISIONS TO REDUCE CARBON EMISSIONS BY DUOPOLY MANUFACTURERS UNDER CARBON TAX POLICY

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Abstract

To analyze the behavioural strategies of the manufacturers reducing carbon emissions and the conditions of carbon tax for encouraging manufacturers to reduce carbon emissions under consumers are more willing to buy low-carbon products, the evolutionary game model is developed. Analyzing the impact of the consumers low carbon preference, carbon and tax carbon emissions reduction to manufacturers reducing carbon emissions. It found that the evolutionary equilibrium results of the system affected by the cost of unit reducing carbon emissions and the consumer sensitivity of carbon emissions. There is boundary condition of carbon tax rate, and only when the carbon tax rate is greater than boundary condition, the carbon tax policy can effectively encourage manufacturers to reduce carbon emissions. Government should reduce the costs of reducing carbon emissions and increase consumer sensitivity of carbon emissions, and the carbon tax rate must be greater than boundary conditions for promote the reduction of carbon emissions.

Keywords: carbon tax; carbon emissions reduction, duopoly; evolutionary game; low carbon preference

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1. Introduction

With the development of industrialization, global carbon emissions increasingly serious, which leading to a series of environmental problems (Li et al., 2019). In the past few decades, global carbon emissions have risen by 65 percent (Hassiba and Linke, 2017). The key of reducing carbon emissions uses clean energy and materials to producing, the core of which is innovation in production technology. However, they may not reduce carbon emissions for maximize their own benefit, therefore, the behavior of manufacturers about reducing carbon emissions need the guidance of nation government (Chen and Hu, 2018). However, the lack of relevant low-carbon policies will lead manufacturers to reduce the investment of reducing carbon emissions and low-carbon technology innovation for their benefit (Wu et al., 2017). Many countries launched the carbon tax policicly for encouraging manufacturers to reducing carbon emissions. For example Finland, Sweden, Norway, Netherlands, Denmark, Slovenia, Italy, Germany, United Kingdom, Switzerland, United States, etc. (Atasu et al., 2013; Choudhary et al., 2015; Krass et al., 2013). Many scholars have shown that carbon emissions policy can encouraging manufacturers to reduce carbon emissions (Cohen and Vandenbergh, 2012; Kanada et al., 2013; Krass et al., 2013), and carbon tax is one of the these, the government levies a tax on carbon emissions. Many mmanufacturers used low-carbon production technologies to reduce carbon emissions for reducing costs of production (Drake et al., 2016). For example, Apple issued $1.5 billion in bonds for investments of

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reducing carbon emissions in 2011, and as a result, the company reduced carbon emissions by 89,000 ton on 2016 (Shi et al., 2018). However, carbon emissions reduction rise the cost and lead to the reduction in profits. So, many manufacturs reject carbon emissions reduction (Bi et al., 2015; Shuai et al., 2014). At present, China's low-carbon policies lack effectiveness(Wang and Chang, 2018). So, how shoud the Government to lattch the carbon lax to incentive reducing carbon emission.

Market research found that reduction of carbon emissions can increasing demand for products. (Grimmer and Bingham, 2013; Michaud et al., 2013; Vanclay et al., 2011; Zhao and Zhong, 2015). For example, Wal-Mart spent 30 million US dollars to develop a green refrigerator, and its sales volume increased by 20% (Zhao et al., 2016). However, many studies have shown that due to the investment for Low carbon production technology and market risk, manufacturers cannot increased their profits with reducing carbon emissions and manufacturers may not choose to reduce carbon emissions.(Bi et al., 2015; Shuai et al., 2014; Zhao et al., 2013). Manufacturers are facing uncertainty in choosing low-carbon strategy. Government can make carbon tax policy ,and it can rise the cost of carbon emissions (He et al., 2018; Wu et al., 2017). Based on the above analysis, we mainly solved follow problems:

- What conditions can manufacturers reduce carbon emissions?
- How should the government set up carbon tax for encouraging manufacturers reduce carbon emissions?
- What measures should the government take for promoting manufacturers to reduce emissions?

In order to solve the above problems, we consulted relevant literature and found that evolutionary games provide a good analytical framework for analyzing the dynamic evolution of the system (Wang et al., 2014). First, we established an evolutionary game model of the interaction between two manufacturers.

We used this model to explain the equilibrium conditions of manufacturers reducing carbon emissions. Secondly, we established an evolutionary game model of the interaction between two manufacturers under carbon tax policy, and explaining the impact of carbon tax on evolutionary stable strategy (ESS). Finally, we concluded that carbon tax rate must be greater than boundary condition for effectively encouraged manufacturers to reduce carbon emissions, and this result can provide theoretical guidance for the government to develop carbon tax policy.

The remainder of the paper is structured as follows. Section 2 reviews the relevant literature. Section 3 describes the problems and presents basic assumptions of the model as well as provides the model's parameters and variables. Section 4 analysis the payoff matrix of duopoly manufacturers. Section 5 analysis the result of evolutionary for the duopoly manufacturers. Section 6 analysis the payoff matrix of duopoly manufacturers under carton tax policy. Section 7 analysis the result of evolutionary under carton tax policy for the duopoly manufacturers. Section 8 is analysis the evolutionary path of the manufacturers about adopted low-carbon manufacturing with numerical simulations. Section 9 is the discussion. The conclusions and recommendations in Section 9.

2. Literature review

Many scholars have studied the issue of the reducing carbon emissions in supply chain under carbon tax policy. It mainly includes the following topics.

(1) The decision for supply chain under carbon tax policy.

Choi (2013) used ordering and inventory model to analysis the impact of carbon tax on the rapid response system of the garment industry. Lorek and Spangenberg (2014) confirmed the incentives can promote low carbon consumption. Hu et al. (2014) found that Pigou tax and subsidy incentives can promoting manufacturers to reduce carbon emissions, and the characteristics of the product affect the effect of the incentive. Drake et al. (2016) found that carbon tax can encouraging manufacturers to reduce carbon emissions, and carbon tax rate affect the reduction of carbon emissions. It was need manufacturers to invest of adapting equipment and production technology for reducing carbon emissions. Government should provide subsidies to manufacturers for promote manufacturer to reduce carbon emissions (Park, 2014). In addition, the policy of carbon tax and subsidy are combined, which can promote manufacturers remanufacturing (Saxena et al., 2018). Cohen et al. (2015) found that government subsidies is affect the pricing of suppliers, and which can improve consumer surplus. If only have the subsidy policy, the economic and environmental goals can been achieved when set great subsidy method (Niu et al., 2017). Theißen and Spinler (2014) found that supply chain members can getting more profits with co-investing of reducing carbon emissions, and the contract with two stage pricing (Swami and Shah, 2013), the investment of retailers to manufacturers (Toptal et al., 2014), the contract with supply chain members sharing the cost of reducing carbon emissions (Ghosh and Shah, 2015), which can coordinate low carbon supply chain. Cao et al. (2020) studied the remanufacturing decisions under the carbon tax policy, the carbon tax rise the remanufacture product quantity. In addition, the carbon tax can lead to batter environmental sustainability (Dou and Cao, 2020). Ding et al. (2020) examined the impact of carbon tax and take-back legislation on the production and carbon emission reduction, the manufacturer more willing to carbon emission reduction in the remanufacturing, and the carbon tax leads to the manufacturer more willing to remanufacturing.

(2) The decision for supply chain under the

carbon tax policy and considers the low carbon preferences of consumers.

With the consumers become more conscious of protect environment, consumers are willing to buy low-carbon products (Michaud et al., 2013; Vanclay et al., 2011). Therefore, low carbon preference of consumers has become one of the important factors for decision of supply chain (Du et al., 2015; Liu et al., 2012). Li (2018) found that retailers' investment decisions on emission reduction are related to the cost of carbon reduction/tax rate/ low carbon preferences of consumer.Zhang and Liu (2013) researched that the market demand is related to carbon emissions, using revenue sharing contract to coordination supply chain. Du et al. (2015) used a carbon-related price–discount sharing-like scheme to achieve joint reduction of carbon emissions for supply chain members and coordinate the supply chain after adjusting the contract parameters.

(3) Use of evolutionary game to study low carbon supply chain.

Smith used game theory to analysis the evolution of biology, after that many scholars used it to analysis economic processes (Friedman, 1991; Young, 1993). Among them in the application of low carbon supply chain, Zhao et al. (2016) used the system dynamics simulation method to analyze the dynamic evolution process of manufacturers and governments. It found that the combining subsidy and carbon tax is more efficient than the carton tax. Tian et al. (2014) used the evolutionary game model to prove that the subsidy manufacturers with reduction of carbon emissions is more conducive to reducing carbon emissions than subsidy the consumers. Wu et al. (2017) established an evolutionary game model between enterprises and governments under different carbon emissions policies.

It was found that enterprises hope government incentives (subsidies or carbon regulations), and incentives can accelerate the evolution of the system. In addition, the subsidy policies are most effective incentives. Chen and Hu (2018) established an evolutionary game model between government and manufacturers. It found that dynamic tax and dynamic subsidies policy is most effectively for manufacturers to adopt low carbon strategies.

The government should also optimize policies of carbon emissions based on the adjustment of the low-carbon strategy with manufacturer. In summary, scholars have studied the low-carbon supply chain and achieved rich results. Based on the existing research results, our key contribution lies in the following several aspects:

1) We considered low carbon preferences of consumers and the substitution of low carbon products and ordinary products.

2). This paper considered the two-population competition and studied the boundary of manufacturer reduction carbon emission.

3) This paper considered the manufacturer limited rationality.

3. Problem description and assumptions

3.1. Problem description

Low-carbon production refers to reducing carbon emissions in the production process through technological reforms, improving resource utilization and reducing energy consumption (Chen and Hu, 2018). Many manufacturers may not reduce carbon emissions for to reduce the cost of carbon emissions reduction. In addition, relevant studies have found that low-carbon preference behavior of consumers are increase the demand for low-carbon products, and improving the earnings of manufacturer (Liu et al., 2012; Michaud et al., 2013). Thus, it is a game problem related to decision-making with manufacturers whether adopt low-carbon technology or not. In addition, how should the government set up a carbon tax policy for encourage manufacturer to adopt low carbon strategy?

This paper studies two undifferentiated and competitive oligopoly manufacturers (Manufacturers 1 and 2). There find great behavioral strategies through constant learning and trial-and-error gradually. Manufacturers make production and sales decisions, and consider whether to make reduction of carbon decisions (D indicates reduction of carbon emissions, and N indicates without reduction of carbon emissions).

3.2. Model assumptions and parameters and variables symbol descriptions

In order to facilitate the analysis, this paper makes the following assumptions:

Assumption 1: Low-carbon product 2 is produced by low-carbon production technology, and ordinary product 1 is produced by ordinary production technology. The functions of the two products are same. Referring to (Liu et al., 2012; Zhang et al., 2015), we assume that the market demand function of the product is:

$q_i = a - p_i + \theta p_j - \tau(e_i - \theta e_j),\ i,j = 1,2,$

where a is the potential demand of the market, $\theta (0<\theta<1)$ indicates the substitution factor of the product 1 and 2, τ indicates consumer sensitivity of carbon emissions, e indicates the carbon emissions of per unit product.

Assumption 2: The carbon emissions of unit production when the manufacturer reduction (without reduction) of carbon emissions or reduction of carbon emissions is e_1 (e_0)Tian et al. (2014).

Assumption 3: Referring to Chen and Hu (2018), we assume that the ordinary/ low-carbon product production costs of unit produce is c_1/c_2.

The parameter and variable symbols and their meanings are shown in Tables 1 and 2.

Table 1. Comparison of this paper with related literature

Literature	***Evolutionary game***	***low carbon preferences***	***Carbon tax***	***Carbon emission reduction strategy***	***The boundary of carbon tax***
Cao (2020); Ding et al. (2020); Drake et al. (2016)	NO	NO	YES	YES	NO
Zhang and Liu (2013)	NO	YES	NO	YES	NO
Du et al. (2015)	NO	YES	YES	YES	NO
Tian et al. (2014)	YES	NO	NO	YES	NO
Wu et al. (2017); Chen and Hu (2018)	YES	NO	YES	YES	NO
This paper	YES	YES	YES	YES	YES

Table 2. Parameters and variables symbol descriptions

Parameters and variables symbol	***Descriptions***
c_t	Carbon tax rate
$p_i^{\alpha,\beta}$	When the strategy combination is (α,β), the product price of the manufacturer i
$q_i^{\alpha,\beta}$	When the strategy combination is (α,β), the product quantity of the manufacturer i
$\pi_i^{\alpha,\beta}$	When the strategy combination is (α,β), the profit of the manufacturer i
$p_{iT}^{\alpha,\beta}$	When the strategy combination is (α,β), the product price of the manufacturer i under carbon tax policy
$q_{iT}^{\alpha,\beta}$	When the strategy combination is (α,β), the product quantity of the manufacturer i under carbon tax policy
$\pi_i^{\alpha,\beta}$	When the strategy combination is (α,β), the profit of the manufacturer i under carbon tax policy

The $i=1,2$; $\alpha,\beta = D,N$

4. Payoff matrix analysis of duopoly manufacturers

The duopoly manufacturers can choose to reduce carbon emissions or without reduce carbon emissions. According to the different strategy, the analyze is as following:

(1) The duopoly manufacturers strategy combination is (D, D), which is the manufacturer 1 and the manufacturer 2 choose to reduce carbon emissions. Due to there are undifferentiated between the manufacturer 1 and the manufacturer 2, the carbon emission of them is equal (Liu et al., 2012). The decision function of duopoly manufacturers are as follows (Eq. 1):

$$Max\pi_i^{DD} = (p_i^{DD} - c_2)q_i^{DD}, i = 1,2 \tag{1}$$

The optimal results are as given by Eqs. (1-4):

$$p_i^{DD*} = \frac{a + c_2 - (1-\theta)\tau e_1}{2-\theta} \tag{2}$$

$$q_i^{DD*} = \frac{a - (1-\theta)(c_2 + e_1\tau)}{2-\theta} \tag{3}$$

$$\pi_i^{DD*} = \{\frac{a - (1-\theta)(c_2 + e_1\tau)}{2-\theta}\}^2, i = 1,2 \tag{4}$$

(2) The duopoly manufacturers strategy combination is (N, D), which is the manufacturer 2 choose to reduce carbon emissions, but the manufacturer 1 choose to without reduction of carbon emissions. The manufacturer 1 decision function is as expressed by Eq. (5).

$$Max\pi_1^{ND} = (p_1^{ND} - c_1)q_1^{ND} \tag{5}$$

The manufacturer 2 decision function is as give by Eq. (6).

$$Max\pi_2^{ND} = (p_2^{ND} - c_2)q_2^{ND} \tag{6}$$

The optimal results are as expressed by Eqs. (7-12):

$$p_1^{ND*} = \frac{(2+\theta)a + 2c_1 + \theta c_2 - \tau[(2-\theta^2)e_0 - \theta e_1]}{4-\theta^2} \tag{7}$$

$$p_2^{ND*} = \frac{(2+\theta)a + 2c_2 + \theta c_1 - \tau[(2-\theta^2)e_1 - \theta e_0]}{4-\theta^2} \tag{8}$$

$$q_1^{ND*} = \frac{(2+\theta)a - (2-\theta^2)(c_1 + \tau e_0) + \theta(c_2 + \tau e_1)}{4-\theta^2} \tag{9}$$

$$q_2^{ND*} = \frac{(2+\theta)a - (2-\theta^2)(c_2 + \tau e_1) + \theta(c_1 + \tau e_0)}{4-\theta^2} \tag{10}$$

$$\pi_1^{ND*} = [\frac{(2+\theta)a - (2-\theta^2)(c_1 + \tau e_0) + \theta(c_2 + \tau e_1)}{4-\theta^2}]^2 \tag{11}$$

$$\pi_2^{ND*} = [\frac{(2+\theta)a-(2-\theta^2)(c_2+\tau e_1)+\theta(c_1+\tau e_0)}{4-\theta^2}]^2 \quad (12)$$

(3) The duopoly manufacturers strategy combination is (D, N), which is the manufacturer 1 choose to reduce carbon emissions, but the manufacturer 2 choose to without reduction of carbon emissions. It is similar to the situation (2). It only needs to exchange the decisions of two manufacturers in situation (2), so the optimal profits of the two manufacturers are as given by Eqs. (13, 14):

$$\pi_1^{DN*} = [\frac{(2+\theta)a-(2-\theta^2)(c_2+\tau e_1)+\theta(c_1+\tau e_0)}{4-\theta^2}]^2 \quad (13)$$

$$\pi_2^{DN*} = [\frac{(2+\theta)a-(2-\theta^2)(c_1+\tau e_0)+\theta(c_2+\tau e_1)}{4-\theta^2}]^2 \quad (14)$$

(4) The duopoly manufacturers strategy combination is (N, N), which is the manufacturer 1 and 2 choose to without reduction of carbon emissions. The decision function of duopoly manufacturers are as follows (Eq. 15):

$$Max\pi_i^{NN} = (p_i^{NN}-c_1)q_i^{NN}, i=1,2 \quad (15)$$

The optimal results are as given by Eqs. (16-18):

$$p_i^{NN*} = \frac{a+c_1-(1-\theta)\tau e_0}{2-\theta} \quad (16)$$

$$q_i^{NN*} = \frac{a-(1-\theta)(c_1+\tau e_0)}{2-\theta} \quad (17)$$

$$\pi_i^{NN*} = [\frac{a-(1-\theta)(c_1+\tau e_0)}{2-\theta}]^2, i=1,2 \quad (18)$$

According to the above model analysis, we can get the payoff matrix between duopoly manufacturers, as shown in Table 3.

Table 3. Payoff matrix between the manufacturers 1 and 2

		Manufacturer 2	
		Reduce carbon emissions (D)	***Not reduce carbon emissions (N)***
Manufacturer 1	***Reduce carbon emissions (D)***	$(\pi_1^{DD*}, \pi_2^{DD*})$	$(\pi_1^{DN*}, \pi_2^{DN*})$
	Not reduce carbon emissions (N)	$(\pi_1^{ND*}, \pi_2^{ND*})$	$(\pi_1^{NN*}, \pi_2^{NN*})$

5. Evolutionary game analysis of duopoly manufacturers reducing carbon emissions decision

5.1. Equilibrium points of the evolutionary process

Suppose the ratio of manufacturer 1/2 choosing reduction of carbon emissions strategy (*D*) is x/y ($x, y \in [0,1]$), and the ratio of manufacturer 1/2 choose to without reduction of carbon emissions strategy (*N*) is $1-x/1-y$.

The payoffs of the manufacturer 1 with choosing reduction of carbon emissions strategy is as follows (Eq. 19):

$$U_{1D} = y\pi_1^{DD*} + (1-y)\pi_1^{DN*} \quad (19)$$

The payoffs of the manufacturer 1 with choosing without reduction of carbon emissions strategy is as follows (Eq. 20):

$$U_{1N} = y\pi_1^{ND*} + (1-y)\pi_1^{NN*} \quad (20)$$

The average earning of the manufacturer 1 is as following (Eq. 21):

$$\bar{U}_1 = xU_{1D} + (1-x)U_{1N} \quad (21)$$

In the same way, the payoffs and average earning of the manufacturer 2 with the two different behavior strategies are as Eqs. (22-24):

$$U_{2D} = x\pi_2^{DD*} + (1-x)\pi_2^{ND*} \quad (22)$$

$$U_{2N} = x\pi_2^{DN*} + (1-x)\pi_2^{NN*} \quad (23)$$

$$\bar{U}_2 = yU_{2D} + (1-y)U_{2N} \quad (24)$$

According to replicator dynamic equation formula of evolutionary game, we can get the replicator dynamic equation of "reduction of carbon emissions" chosen by manufacturer 1 is expressed by Eq. (25):

$$\frac{dx}{dt} = x(U_{1D}-\bar{U}_1) = x(1-x)[y(\pi_1^{DD*}-\pi_1^{ND*})+(1-y)(\pi_1^{DN*}-\pi_1^{NN*})] \quad (25)$$

The replicator dynamic equation of "reduction of carbon emissions" chosen by manufacturer 2 is as following (Eq. 26):

$$\frac{dy}{dt} = y(U_{2D}-\bar{U}_2) = y(1-y)[x(\pi_2^{DD*}-\pi_2^{DN*})+(1-x)(\pi_2^{ND*}-\pi_2^{NN*})] \quad (26)$$

The replicator dynamic system (I) is treated as the combining of Eq. (25) and Eq. (26) (Eq. 27):

$$\begin{cases} X = \frac{dx}{dt} = x(1-x)[y(\pi_1^{DD*}-\pi_1^{ND*})+(1-y)(\pi_1^{DN*}-\pi_1^{NN*})] \\ Y = \frac{dy}{dt} = y(1-y)[x(\pi_2^{DD*}-\pi_2^{DN*})+(1-x)(\pi_2^{ND*}-\pi_2^{NN*})] \end{cases} \quad (27)$$

Proposition 1. The equilibrium points of the replicator dynamic system (I) are (0, 0), (0, 1), (1, 0), (1, 1). If $(\pi_1^{DD*}-\pi_1^{ND*})(\pi_1^{DN*}-\pi_1^{NN*})<0$ and $(\pi_2^{DD*}-\pi_2^{DN*})(\pi_2^{ND*}-\pi_2^{NN*})<0$, ($x^*$, y^*) is also the equilibrium point of system (I).

Eq. (28) gives the following dependence:

$$x^* = \frac{\pi_2^{NN*} - \pi_2^{ND*}}{\pi_2^{DD*} + \pi_2^{NN*} - \pi_2^{DN*} - \pi_2^{ND*}},\ y^* = \frac{\pi_1^{NN*} - \pi_1^{DN*}}{\pi_1^{DD*} + \pi_1^{NN*} - \pi_1^{DN*} - \pi_1^{ND*}} \tag{28}$$

Proof 1. For system (I), let the replicator dynamic system (I) be $\frac{dx}{dt}=0, \frac{dy}{dt}=0$, we can get $x=0$ or 1, $y=0$ or 1. So the (0, 0), (0, 1), (1, 0), (1, 1) are the equilibrium point of system (I).

When $(\pi_1^{DD*} - \pi_1^{ND*})(\pi_1^{DN*} - \pi_1^{NN*}) > 0$, we can get $y(\pi_1^{DD*} - \pi_1^{ND*}) + (1-y)(\pi_1^{DN*} - \pi_1^{NN*}) \neq 0$ easily.

When $(\pi_2^{DD*} - \pi_2^{DN*})(\pi_2^{ND*} - \pi_2^{NN*}) > 0$, we can get $x(\pi_2^{DD*} - \pi_2^{DN*}) + (1-x)(\pi_2^{ND*} - \pi_2^{NN*}) \neq 0$ easily. So (x^*, y^*) is not the equilibrium point of system (I).

When $(\pi_1^{DD*} - \pi_1^{ND*})(\pi_1^{DN*} - \pi_1^{NN*}) < 0$ and $(\pi_2^{DD*} - \pi_2^{DN*})(\pi_2^{ND*} - \pi_2^{NN*}) < 0$, we can get $y(\pi_1^{DD*} - \pi_1^{ND*}) + (1-y)(\pi_1^{DN*} - \pi_1^{NN*}) = 0$ and $x(\pi_2^{DD*} - \pi_2^{DN*}) + (1-x)(\pi_2^{ND*} - \pi_2^{NN*}) = 0$. Let the replicator dynamic system (I) be $\frac{dx}{dt}=0, \frac{dy}{dt}=0$, we can get ($x^*$, y^*).

5.2. Stability analysis of the equilibrium point

According to Friedman (1991), Young (1993), the stability of the equilibrium points can be analyzed using Jaconbian Matrix and we can get the Evolutionary Stable Strategy (ESS). The Jaconbian Matrix of replicator dynamic system (I) is got by finding first-order partial derivatives of x and y respectively for Eq. (27), which is as Eqs. (29-33):

$$J = \begin{bmatrix} \partial X/\partial x & \partial X/\partial y \\ \partial Y/\partial x & \partial Y/\partial y \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{29}$$

where:

$$a_{11} = (1-2x)[y(\pi_1^{DD*} - \pi_1^{ND*}) + (1-y)(\pi_1^{DN*} - \pi_1^{NN*})] \tag{30}$$

$$a_{12} = x(1-x)(\pi_1^{DD*} - \pi_1^{ND*} - \pi_1^{DN*} + \pi_1^{NN*}) \tag{31}$$

$$a_{21} = y(1-y)(\pi_2^{DD*} - \pi_2^{DN*} - \pi_2^{ND*} + \pi_2^{NN*}) \tag{32}$$

$$a_{22} = (1-2y)[x(\pi_2^{DD*} - \pi_2^{DN*}) + (1-x)(\pi_2^{ND*} - \pi_2^{NN*})] \tag{33}$$

When the Jaconbian Matrix of replicator dynamic system (I) satisfies the condition $a_{11} + a_{22} < 0$ (Trace condition, the value of which is recorded trJ) and $|J| = a_{11}a_{22} - a_{12}a_{21} > 0$ (Determinant condition, the value of which is recorded detJ), the equilibrium point is a ESS (Friedman, 1991; Young, 1993). The trJ and detJ of the equilibrium points show in Table 4.

Eq. (34) gives the following dependence:

$$A = -\frac{(\pi_2^{NN*} - \pi_2^{ND*})(\pi_2^{DD*} - \pi_2^{DN*})(\pi_1^{NN*} - \pi_1^{DN*})(\pi_1^{DD*} - \pi_1^{ND*})}{(\pi_2^{NN*} - \pi_2^{ND*} + \pi_2^{DD*} - \pi_2^{DN*})(\pi_1^{NN*} - \pi_1^{DN*} + \pi_1^{DD*} - \pi_1^{ND*})} \tag{34}$$

According to Table 4, we find that the traces and determinants of each equilibrium points are composed of $\pi_1^{DN*} - \pi_1^{NN*}$, $\pi_1^{DD*} - \pi_1^{ND*}$, $\pi_2^{ND*} - \pi_2^{NN*}$, $\pi_2^{DD*} - \pi_2^{DN*}$. Therefore, we can get the value be as Eqs. (35, 36):

$$\pi_1^{DN*} - \pi_1^{NN*} = \pi_2^{ND*} - \pi_2^{NN*} = \{\frac{(2-\theta^2)(c_1 - c_2 + \tau(e_0 - e_1))}{4-\theta^2}\} \cdot \{\frac{(4+2\theta)a - (2-\theta^2)[c_1 + c_2 + \tau(e_1 + e_0)] + 2\theta(c_1 + e_0\tau)}{4-\theta^2}\} \tag{35}$$

$$\pi_1^{DD*} - \pi_1^{ND*} = \pi_2^{DD*} - \pi_2^{DN*} = \{\frac{(2-\theta^2)[c_1 - c_2 + \tau(e_0 - e_1)]}{4-\theta^2}\} \cdot \{\frac{(4+2\theta)a - (2-\theta^2)[c_1 + c_2 + \tau(e_1 + e_0)] + 2\theta(c_2 + e_1\tau)}{4-\theta^2}\} \tag{36}$$

We find that the values of trJ and detJ in Table 3 are determined by the Eq. (35), Eq. (36) and Eq. (35)-Eq. (36).

According to the assumption 1, we can know $0 < \theta < 1$. Therefore, the value of Eq. (35)-Eq. (36) is as following (Eq. 37):

$$\frac{2\theta(2-\theta^2)[c_1 - c_2 + \tau(e_0 - e_1)]^2}{(4-\theta^2)^2} > 0 \tag{37}$$

We can easily find that be as Eqs. (38, 39):

$$\frac{(4+2\theta)a - (2-\theta^2)[c_1 + c_2 + \tau(e_1 + e_0)] + 2\theta(c_1 + e_0\tau)}{4-\theta^2} = q_1^{DN*} + q_1^{NN*} > 0 \tag{38}$$

$$\frac{(4+2\theta)a - (2-\theta^2)[c_1 + c_2 + \tau(e_1 + e_0)] + 2\theta(c_2 + e_1\tau)}{4-\theta^2} = q_1^{DD*} + q_1^{ND*} > 0 \tag{39}$$

Therefore, we can find that the sign of formula in Table 3 are determined by (Eq. 40):

$$\frac{(2-\theta^2)[c_1 - c_2 + \tau(e_0 - e_1)]}{4-\theta^2} \tag{40}$$

Then, according to the assumption 1, we can know $0 < \theta < 1$. Therefore $2-\theta^2 > 0$, $4-\theta^2 > 0$. When $c_1 - c_2 + \tau(e_0 - e_1) > 0$, we can get $c_2 - c_1 < \tau(e_0 - e_1)$. We can analyze the ESS of the system (I) in two cases $\frac{c_2 - c_1}{e_0 - e_1} < \tau$ (condition I) and $\frac{c_2 - c_1}{e_0 - e_1} > \tau$ (condition II). The results show in Table 5.

Table 4. The trJ and detJ of the equilibrium points

Equilibrium points	***detJ***	***trJ***
(0,0)	$(\pi_1^{DN*}-\pi_1^{NN*})(\pi_2^{ND*}-\pi_2^{NN*})$	$\pi_1^{DN*}-\pi_1^{NN*}+\pi_2^{ND*}-\pi_2^{NN*}$
(0,1)	$(\pi_1^{DD*}-\pi_1^{ND*})(\pi_2^{NN*}-\pi_2^{ND*})$	$\pi_1^{DD*}-\pi_1^{ND*}+\pi_2^{NN*}-\pi_2^{ND*}$
(1,0)	$(\pi_1^{NN*}-\pi_1^{DN*})(\pi_2^{DD*}-\pi_2^{DN*})$	$\pi_1^{NN*}-\pi_1^{DN*}+\pi_2^{DD*}-\pi_2^{DN*}$
(1,1)	$(\pi_1^{ND*}-\pi_1^{DD*})(\pi_2^{DN*}-\pi_2^{DD*})$	$\pi_1^{ND*}-\pi_1^{DD*}+\pi_2^{DN*}-\pi_2^{DD*}$
$(x^*,\ y^*)$	A	0

Table 5. Equilibrium points and Local stability of system (I)

Point	***Condition I***			***Condition II***		
	detJ	***trJ***	***State***	***detJ***	***trJ***	***State***
(0, 0)	+	+	Instability point	+	-	ESS
(0, 1)	-	-	Saddle point	-	-	Saddle point
(1, 0)	-	-	Saddle point	-	-	Saddle point
(1, 1)	+	-	ESS	+	+	Instability point
(x^*, y^*)	Nonequilibrium point			Nonequilibrium point		

"+" denotes greater than zero, "-" denotes less than zero

5.3. Analysis of Evolution Result

According to the stability analysis of the equilibrium point, we find that there are two kinds of ESS:

(1) According to Table 4, we find that (1,1) is an ESS of system (I), (0, 1), (1, 0) are two Saddle point of system (I), (0, 0)is an Instability point of system (I) when $\frac{c_2-c_1}{e_0-e_1}<\tau$. It shows that system (I) will eventually evolve into (1,1). The manufacturers will eventually choose to reduce carbon emissions. $\frac{c_2-c_1}{e_0-e_1}$ is represents the cost of reducing unit carbon emission. Only when the cost of reducing unit carbon emissions is less than τ , all manufacturers of market will reduce carbon emissions. The evolutionary path is displayed in Fig. 1.

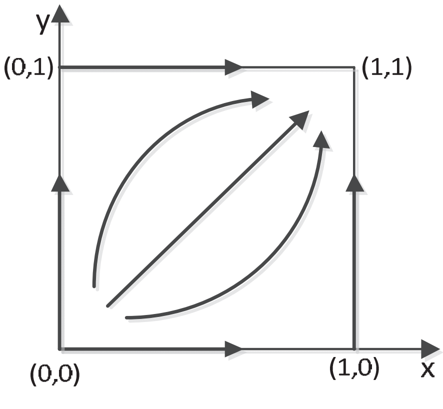

Fig. 1. The evolutionary path of condition I

(2) We find that there are an ESS (0, 0), two saddle point (0, 1) and (1, 0), an instability point (1, 1) of system (I) when $\frac{c_2-c_1}{e_0-e_1}>\tau$. $\frac{c_2-c_1}{e_0-e_1}>\tau$ is indicates that the cost of reducing unit carbon emissions, which is greater than τ. It shows that system (I) will eventually evolve into (0, 0).

The manufacturers will eventually choose to without reduction of carbon emissions, and this is not conducive for environmental protection. The evolutionary path is displayed in Fig. 2.

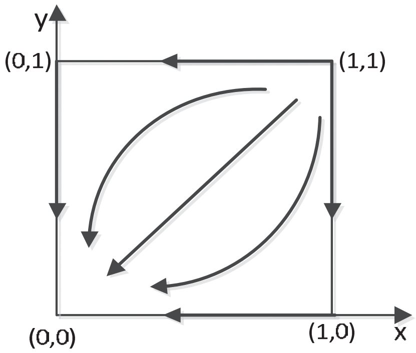

Fig. 2. The evolutionary path of condition II

Proposition 2. Increasing the carbon emissions sensitivity coefficient of consumers can promote the equilibrium point of the system evolve into reducing the carbon emissions of manufacturers.

Proof 2. From the above analysis, we know only if when $\frac{c_2-c_1}{e_0-e_1}<\tau$,the reduce carbon emissions is the ESS of system (I).Therefore, raising τ can make $\tau>\frac{c_2-c_1}{e_0-e_1}$ satisfying easier.

Proposition 2 description. The consumer sensitivity coefficient of carbon emissions is greater, the manufacturer more likely choose to reduce carbon emissions. The government should increase the investment in environmental protection propaganda for raise awareness of energy conservation and environmental protection of consumers. It is conducive to promoting manufacturer reducing carbon emissions.

Proposition 3. Reducing cost of reducing carbon emissions $(c_2 - c_1)$ can promote the equilibrium point of the system evolve into reducing the carbon emissions of manufacturers.

Proof 3. From the above analysis, we know only if when $c_2 - c_1 < \tau(e_0 - e_1)$, the ESS of system (I) is manufacturer choose reducing carbon emissions. Therefore, reducing cost of reducing carbon emissions $(c_2 - c_1)$ can make $c_2 - c_1 < \tau(e_0 - e_1)$ satisfying easier.

Proposition 3 description. Reducing the cost of reducing carbon emissions can increase the profit of manufacturer with reducing carbon emissions. In order to increase profits, manufacturer will choose to reduce carbon emissions. The government should increase investment in scientific research, improve the innovation ability of country, provide better production technology and lower cost of production materials, and which can reduce the cost of reducing carbon emissions.

6. Payoff matrix analysis of duopoly manufacturer under carbon tax policy

From above analysis, we can find that only when the cost of reducing unit carbon emissions is less than consumer sensitivity coefficient of carbon emissions ($\frac{c_2 - c_1}{e_0 - e_1} < \tau$), the evolution result is the manufacturers choose to reducing carbon emissions and the social benefits are optimal. Otherwise the manufacturers are not to reduction the carbon emissions, the social benefits are worst. According to Propositions 2 and 3, we can see that increasing the consumer sensitivity coefficient of carbon emissions or reducing the cost of reduction carbon emissions can promote manufacturers choose to reduction the carbon emissions. However, the above behavior is difficult to implementation in a short time. Therefore, the government can use carbon tax policies to promote manufacturer reducing carbon emissions. The relevant literature has studied the impact of carbon tax policies on manufacturer reducing carbon emissions (Atasu et al., 2013; Drake et al., 2016; Du et al., 2017; Saxena et al., 2018). In this context, we use carbon tax policy into evolutionary game framework, and analyzes how the government should set carbon tax rate for the evolutionary stable strategy is manufacturers choose to reducing carbon emissions.

Under the carbon tax policy, the profits analysis of different strategic combinations of duopoly manufacturer is as follows:

(1) The duopoly manufacturer's strategy combination is (D, D). The decision function of duopoly manufacturers are as follows (Eq. 40):

$$Max\pi_{iT}^{DD} = (p_{iT}^{DD} - c_2 - c_t e_1)q_{iT}^{DD}, i = 1,2 \tag{41}$$

The optimal result are as Eqs. (42-44):

$$p_{iT}^{DD*} = \frac{a + c_2 + e_1[c_t - \tau(1-\theta)]}{2-\theta} \tag{42}$$

$$q_{iT}^{DD*} = \frac{a - (1-\theta)[c_2 + e_1(c_t + \tau))]}{2-\theta} \tag{43}$$

$$\pi_{iT}^{DD*} = \{\frac{a - (1-\theta)[c_2 + e_1(c_t + \tau)]}{2-\theta}\}^2, i = 1,2 \tag{44}$$

(2) The duopoly manufacturer's strategy combination is (N, D), and the manufacturer 1 decision function is as follows (Eq. 45):

$$Max\pi_{1T}^{ND} = (p_{1T}^{ND} - c_1 - c_t e_0)q_{1T}^{ND} \tag{45}$$

The manufacturer 2 decision function is as follows (Eq. 46):

$$Max\pi_{2T}^{ND} = (p_{2T}^{ND} - c_2 - c_t e_1)q_{2T}^{ND} \tag{46}$$

The optimal result are as Eqs. (47-52):

$$p_{1T}^{ND*} = \frac{(2+\theta)a + 2c_1 + \theta c_2 + c_t(2e_0 + \theta e_1) - \tau[(2-\theta^2)e_0 - \theta e_1]}{4-\theta^2} \tag{47}$$

$$p_{2T}^{ND*} = \frac{(2+\theta)a + 2c_2 + \theta c_1 + c_t(2e_1 + \theta e_0) - \tau[(2-\theta^2)e_1 - \theta e_0]}{4-\theta^2} \tag{48}$$

$$q_{1T}^{ND*} = \frac{(2+\theta)a - (2-\theta^2)[c_1 + (c_t + \tau)e_0] + \theta[c_2 + (c_t + \tau)e_1]}{4-\theta^2} \tag{49}$$

$$q_{2T}^{ND*} = \frac{(2+\theta)a - (2-\theta^2)[c_2 + (c_t + \tau)e_1] + \theta[c_1 + (c_t + \tau)e_0]}{4-\theta^2} \tag{50}$$

$$\pi_{1T}^{ND*} = \{\frac{(2+\theta)a - (2-\theta^2)[c_1 + (c_t + \tau)e_0] + \theta[c_2 + (c_t + \tau)e_1]}{4-\theta^2}\}^2 \tag{51}$$

$$\pi_{2T}^{ND*} = \{\frac{(2+\theta)a - (2-\theta^2)[c_2 + (c_t + \tau)e_1] + \theta[c_1 + (c_t + \tau)e_0]}{4-\theta^2}\}^2 \tag{52}$$

(3) The duopoly manufacturers strategy combination is (D, N), and the optimal profits of the two manufacturers are as Eqs. (53, 54):

$$\pi_{1T}^{DN*} = \{\frac{(2+\theta)a - (2-\theta^2)[c_2 + (c_t + \tau)e_1] + \theta[c_1 + (c_t + \tau)e_0]}{4-\theta^2}\}^2 \tag{53}$$

$$\pi_{2T}^{DN*} = \{\frac{(2+\theta)a-(2-\theta^2)[c_1+(c_t+\tau)e_0]+\theta[c_2+(c_t+\tau)e_1]}{4-\theta^2}\}^2 \quad (54)$$

(4) The duopoly manufacturer's strategy combination is (N, N). The decision function of duopoly manufacturers are as follows (Eq. 55):

$$Max\pi_{iT}^{NN} = (p_{iT}^{NN} - c_1 - c_t e_0)q_{iT}^{NN} \quad (55)$$

The optimal results are as Eqs. (56-58):

$$p_{iT}^{\mathrm{NN}*} = \frac{a+c_1+e_0[c_t-\tau(1-\theta)]}{2-\theta} \quad (56)$$

$$q_{iT}^{NN*} = \frac{a-(1-\theta)[c_1+e_0(c_t+\tau))]}{2-\theta} \quad (57)$$

$$\pi_{iT}^{NN*} = \{\frac{a-(1-\theta)[c_1+e_0(c_t+\tau)]}{2-\theta}\}^2 \quad (58)$$

According to the above model analysis, we can get the payoff matrix between duopoly manufacturers under carbon tax policy, as shown in Table 6.

7. Evolutionary game analysis of duopoly manufacturers reducing carbon emissions decision under carbon tax policy

7.1. Equilibrium points of the evolutionary process

According to the way of replicator dynamic system (I), we can get the replicator dynamic system under carbon tax policy (I_T) (Eq. 59).

$$\begin{cases} X_T = \frac{dx}{dt} = x(1-x)[y(\pi_{1T}^{DD*}-\pi_{1T}^{ND*})+(1-y)(\pi_{1T}^{DN*}-\pi_{1T}^{NN*})] \\ Y_T = \frac{dy}{dt} = y(1-y)[x(\pi_{2T}^{DD*}-\pi_{2T}^{DN*})+(1-x)(\pi_{2T}^{ND*}-\pi_{2T}^{NN*})] \end{cases}$$

(59)

Proposition 4. The equilibrium points of the replicator dynamic system (I_T) are (0, 0), (0, 1), (1, 0), (1, 1). If $(\pi_{1T}^{DD*}-\pi_{1T}^{ND*})(\pi_{1T}^{DN*}-\pi_{1T}^{NN*})<0$ and $(\pi_{2T}^{DD*}-\pi_{2T}^{DN*})(\pi_{2T}^{ND*}-\pi_{2T}^{NN*})<0$, ($x_T^*$, y_T^*) is also the equilibrium point of system (I_T).

Where Eqs. (60, 61):

$$x_T^* = \frac{\pi_{2T}^{NN*}-\pi_{2T}^{ND*}}{\pi_{2T}^{DD*}+\pi_{2T}^{NN*}-\pi_{2T}^{DN*}-\pi_{2T}^{ND*}} \quad (60)$$

$$y_T^* = \frac{\pi_{1T}^{NN*}-\pi_{1T}^{DN*}}{\pi_{1T}^{DD*}+\pi_{1T}^{NN*}-\pi_{1T}^{DN*}-\pi_{1T}^{ND*}} \quad (61)$$

Proof 4. It is same as proposition 1.

7.2. Stability analysis of the equilibrium point

The Jaconbian Matrix of replicator dynamic system (I_T) is got by finding first-order partial derivatives of x and y respectively for Eq. (21), which is as following (Eq. 62):

$$J_T = \begin{bmatrix} \partial X_T/\partial x & \partial X_T/\partial y \\ \partial Y_T/\partial x & \partial Y_T/\partial y \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad (62)$$

Eqs. (63-66) give the following dependences:

$$a_{11} = (1-2x)[y(\pi_{1T}^{DD*}-\pi_{1T}^{ND*})+(1-y)(\pi_{1T}^{DN*}-\pi_{1T}^{NN*})] \quad (63)$$

$$a_{12} = x(1-x)(\pi_{1T}^{DD*}-\pi_{1T}^{ND*}-\pi_{1T}^{DN*}+\pi_{1T}^{NN*}) \quad (64)$$

$$a_{21} = y(1-y)(\pi_{2T}^{DD*}-\pi_{2T}^{DN*}-\pi_{2T}^{ND*}+\pi_{2T}^{NN*}) \quad (65)$$

$$a_{22} = (1-2y)[x(\pi_{2T}^{DD*}-\pi_{2T}^{DN*})+(1-x)(\pi_{2T}^{ND*}-\pi_{2T}^{NN*})] \quad (66)$$

The trJ and detJ of the equilibrium points show in Table 7.

Eq. (67) describes the following relationships:

$$A_T = -\frac{(\pi_{2T}^{NN*}-\pi_{2T}^{ND*})(\pi_{2T}^{DD*}-\pi_{2T}^{DN*})(\pi_{1T}^{NN*}-\pi_{1T}^{DN*})(\pi_{1T}^{DD*}-\pi_{1T}^{ND*})}{(\pi_{2T}^{NN*}-\pi_{2T}^{ND*}+\pi_{2T}^{DD*}-\pi_{2T}^{DN*})(\pi_{1T}^{NN*}-\pi_{1T}^{DN*}+\pi_{1T}^{DD*}-\pi_{1T}^{ND*})} \quad (67)$$

According to Table 7, we find that the traces and determinants of each equilibrium points are composed of $\pi_{1T}^{DN*}-\pi_{1T}^{NN*}$, $\pi_{1T}^{DD*}-\pi_{1T}^{ND*}$, $\pi_{2T}^{ND*}-\pi_{2T}^{NN*}$, $\pi_{2T}^{DD*}-\pi_{2T}^{DN*}$. Therefore, we can get the value be as Eqs. (68, 69):

$$\pi_{1T}^{DN*}-\pi_{1T}^{NN*} = \pi_{2T}^{ND*}-\pi_{2T}^{NN*} = \{\frac{(2-\theta^2)[c_1-c_2+(c_t+\tau)(e_0-e_1)]}{4-\theta^2}\}\cdot \{\frac{(4+2\theta)a-(2-\theta^2)[c_1+c_2+(c_t+\tau)(e_1+e_0)]+2\theta[c_1+e_0(c_t+\tau)]}{4-\theta^2}\} \quad (68)$$

$$\pi_{1T}^{DD*}-\pi_{1T}^{ND*} = \pi_{2T}^{DD*}-\pi_{2T}^{DN*} = \{\frac{(2-\theta^2)[c_1-c_2+(c_t+\tau)(e_0-e_1)]}{4-\theta^2}\}\cdot \{\frac{(4+2\theta)a-(2-\theta^2)[c_1+c_2+(c_t+\tau)(e_1+e_0)]+2\theta[c_2+e_1(c_t+\tau)]}{4-\theta^2}\} \quad (69)$$

We find that the values of trJ and detJ in Table 7 are determined by the Eq. (68), Eq. (69) and Eq. (68)-Eq. (69).

According to the assumption 1, we can know $0<\theta<1$. Therefore, the value of Eq. (68)-Eq. (69) is as following (Eq. 70):

$$\frac{2\theta(2-\theta^2)[c_1-c_2+(c_t+\tau)(e_0-e_1)]^2}{(4-\theta^2)^2} > 0 \quad (70)$$

Table 6. Payoff matrix between the manufacturers 1 and 2 under carbon tax policy

		Manufacturer 2	
		Reduce carbon emissions (D)	***Not reduce carbon emissions (N)***
Manufacturer 1	***Reduce carbon emissions (D)***	$(\pi_{1T}^{DD*},\ \pi_{2T}^{DD*})$	$(\pi_{1T}^{DN*},\ \pi_{2T}^{DN*})$
	Not reduce carbon emissions (N)	$(\pi_{1T}^{ND*},\ \pi_{2T}^{ND*})$	$(\pi_{1T}^{NN*},\ \pi_{2T}^{NN*})$

Table 7. The trJ and detJ of the equilibrium points under carbon tax policy

Equilibrium points	***detJ***	***trJ***
(0,0)	$(\pi_{1T}^{DN*}-\pi_{1T}^{NN*})(\pi_{2T}^{ND*}-\pi_{2T}^{NN*})$	$\pi_{1T}^{DN*}-\pi_{1T}^{NN*}+\pi_{2T}^{ND*}-\pi_{2T}^{NN*}$
(0,1)	$(\pi_{1T}^{DD*}-\pi_{1T}^{ND*})(\pi_{2T}^{NN*}-\pi_{2T}^{ND*})$	$\pi_{1T}^{DD*}-\pi_{1T}^{ND*}+\pi_{2T}^{NN*}-\pi_{2T}^{ND*}$
(1,0)	$(\pi_{1T}^{NN*}-\pi_{1T}^{DN*})(\pi_{2T}^{DD*}-\pi_{2T}^{DN*})$	$\pi_{1T}^{NN*}-\pi_{1T}^{DN*}+\pi_{2T}^{DD*}-\pi_{2T}^{DN*}$
(1,1)	$(\pi_{1T}^{ND*}-\pi_{1T}^{DD*})(\pi_{2T}^{DN*}-\pi_{2T}^{DD*})$	$\pi_{1T}^{ND*}-\pi_{1T}^{DD*}+\pi_{2T}^{DN*}-\pi_{2T}^{DD*}$
$(x_T^*,\ y_T^*)$	A_T	0

We can easily find that be as Eqs. (71, 72):

$$\frac{(4+2\theta)a-(2-\theta^2)[c_1+c_2+(c_t+\tau)(e_1+e_0)]+2\theta[c_1+e_0(c_t+\tau)]}{4-\theta^2}=q_{1T}^{DN*}+q_{1T}^{NN*}>0 \tag{71}$$

$$\frac{(4+2\theta)a-(2-\theta^2)[c_1+c_2+(c_t+\tau)(e_1+e_0)]+2\theta[c_2+e_1(c_t+\tau)]}{4-\theta^2}=q_{1T}^{DD*}+q_{1T}^{ND*}>0 \tag{72}$$

Therefore, we can find that the sign of formula in Table 6 is determined by Eq. (73):

$$\frac{(2-\theta^2)[c_1-c_2+(c_t+\tau)(e_0-e_1)]}{4-\theta^2} \tag{73}$$

Then, according to the assumption 1, we can know $0<\theta<1$. Therefore $2-\theta^2>0$, $4-\theta^2>0$. When $c_1-c_2+(c_t+\tau)(e_0-e_1)>0$, we can get $\frac{c_2-c_1}{e_0-e_1}<c_t+\tau$. We can analyze the ESS of the system I_T in two cases $\frac{c_2-c_1}{e_0-e_1}<c_t+\tau$ (condition III) and $\frac{c_2-c_1}{e_0-e_1}>c_t+\tau$ (condition IV). The results are shown in Table 8.

7.3. Analysis of Evolution Result

According to the stability analysis of the equilibrium point, we find that there are two kinds of ESS:

(1) According to Table 8, we find that there are one ESS (1, 1), two Saddle point (0, 1), (1, 0), one Instability point (0, 0) when $\frac{c_2-c_1}{e_0-e_1}<c_t+\tau$. It shows that system ($I_T$) will eventually evolve into (1, 1), that is the manufacturers will reduce carbon emissions. $\frac{c_2-c_1}{e_0-e_1}$ is represents the cost of reducing unit carbon emissions and only when it is less than $c_t+\tau$, all manufacturers of market will reduce carbon emissions.

Proposition 5. When $c_t>\frac{c_2-c_1}{e_0-e_1}-\tau$, reducing carbon emissions of manufacturer is the ESS of the system (I_T).

Proof 5. From the above analysis, we know only if when $c_2-c_1<(c_t+\tau)(e_0-e_1)$, the ESS of system ($I_T$) is manufacturer choose to reducing carbon emissions. Therefore, we can get $c_t>\frac{c_2-c_1}{e_0-e_1}-\tau$.

Proposition 5 description. The carbon tax rate has boundary condition. Only when the carbon tax rate is greater than the boundary condition, the carbon tax policy can encourage manufacturer to reduce carbon emissions. Therefore, the government should set suitable carbon tax rate for promote to reducing carbon emissions of manufacturer, which is conducive to the development of energy conservation and environmental protection.

(2) We find that there are one ESS (0, 0), two Saddle point (0, 1) and (1, 0), one Instability point (1, 1) when $\frac{c_2-c_1}{e_0-e_1}>c_t+\tau$. $\frac{c_2-c_1}{e_0-e_1}>c_t+\tau$ is indicates that the cost of reducing unit carbon emissions is greater than $c_t+\tau$. It shows that the system of I_T will eventually evolve into (0, 0). The manufacturers will eventually choose not to reduce carbon emissions, and this is not conducive to environmental protection.

Table 8. Equilibrium points and Local stability of system (I_T)

Point	***Condition III***			***Condition IV***		
	detJ	***trJ***	***State***	***detJ***	***trJ***	***State***
(0,0)	+	+	Instability point	+	-	ESS
(0,1)	-	-	Saddle point	-	-	Saddle point
(1,0)	-	-	Saddle point	-	-	Saddle point
(1,1)	+	-	ESS	+	+	Instability point
(x_T^*, y_T^*)	Nonequilibrium point			Nonequilibrium point		

"+" denotes greater than zero, "-" denotes less than zero

8. Case and simulation analysis

8.1. A case study

China Resources Snow Breweries is stablished in 1993, and located in Beijing, which is a national professional producing and marketing beer company. At present, it operates more than 91 breweries in China, including snow beer brand and more than 30 regional brands. In 2017, the total sales volume of it reached 11.189 million liters, accounting for about 26% of the beer market in China (http://www.snowbeer.com.cn/index.php?r=pro/about/recommend). In this paper, a bottle of 500 ml snow-pure beer is selected as the research object. According to the data collected by Chen and Hu (2018), the carbon emissions of producing a bottle of snow-pure beer is 0.42632 kg, suppose the potential monthly demand of snow-pure beer is 1000 bottles. According to the China Carbon Emissions Trading Network, the carbon emissions of high-quality beer industry adopted low-carbon manufacturing is 0.16256 kg/L, thus the carbon emissions of producing a bottle of high-quality snow-pure beer is 0.08128 kg.

From the Chen and Hu (2018), suppose the production cost of a bottle of snow-pure beer is 3CNY, and the production cost of a bottle of snow-pure beer adopted low-carbon manufacturing is 3.03CNY. Suppose the substitution coefficient of the two products is 0.5, and the consumer sensitivity coefficient of carbon emission is 0.06. The initial parameters for the simulation analysis are offered in Table 9.

8.2. Simulation

The above data with replicator dynamic model are solved and analyzed by Mathematics software. Suppose the initial probability of producing with low-carbon is $x = y = 0.5$. Firstly, the replication dynamic system (I) is analyzed under difference consumer sensitivity coefficient of carbon emission ($\tau = 0.06, 0.065, 0.07, 0.075, 0.08, 0.085, 0.09, 0.095$), and the evolutionary path of the manufacturers about adopted low-carbon manufacturing is solved as shown in Fig. 3.

From it we can find that when the consumer sensitivity coefficient of carbon emission is small, that is to say, when the condition (II) is satisfied, the evolution result of the system is that manufacturers choose to without reduction of carbon emissions. When the consumer sensitivity coefficient of carbon emission is large, that is to say, when the condition (I) is satisfied, the evolution result of the system is that the manufacturer choosing to reduction of carbon emissions, and as consumer sensitivity of carbon emission increases, the system evolves faster. This simulation results verify Proposition 2.

Then, the replication dynamic system (I) is analyzed under difference production cost of produce adopted low-carbon manufacturing ($c_2 = 3.01, 3.015, 3.02, 3.025, 3.03$), and the evolutionary path of the manufacturers about adopted low-carbon manufacturing is solved as shown in Fig. 4. From it we can find that when the production cost of produce adopted low-carbon manufacturing is high, the evolution result of the system is that manufacturers choose to without reduction of carbon emissions.

When the production cost of produce adopted low-carbon manufacturing is low, the evolution result of the system is that manufacturers choose to reduction of carbon emissions, and as production cost of low-carbon products decreases, the system evolves faster. This simulation results verify Proposition 3.

Finally, the replication dynamic system (II) is analyzed under difference carbon tax rate ($c_t = 0.01, 0.015, 0.02, 0.025, 0.03, 0.035$), and the evolutionary path of the manufacturers about adopted low-carbon manufacturing is solved as shown in Fig. 5. From it we can find that when the carbon tax rate is lower than the boundary conditions, the evolution result of the system is that manufacturers choose to without reduction of carbon emissions.

When the carbon tax rate is greater than the boundary conditions, the evolution result of the system is that manufacturers choose to reduction of carbon emissions, and as carbon tax rate increases, the system evolves faster. This simulation results verify Proposition 5.

Table 9. Initial values of the parameters.

Parameters	a	c_1	c_2	e_0	e_1	τ	θ
Values	1000	3	3.03	0.42632	0.08128	0.06	0.5

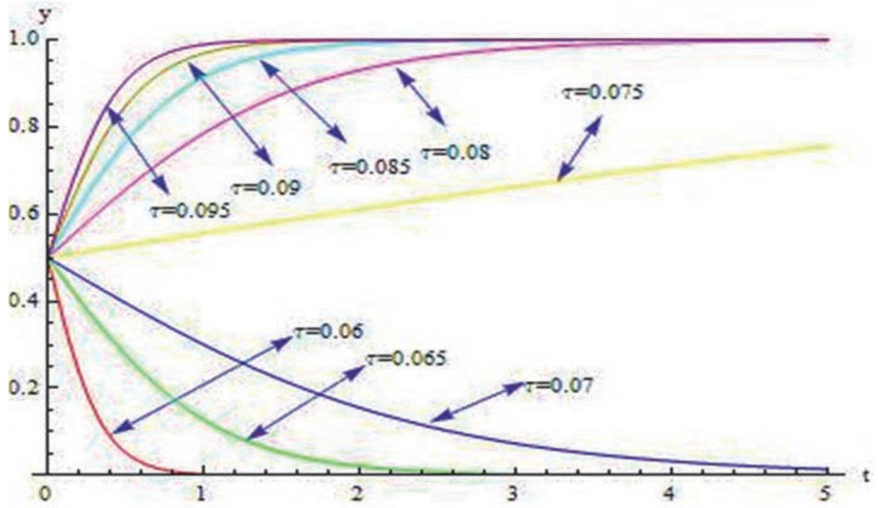

Fig. 3. Evolutionary path of the manufacturers' behavioral strategy in the replicator dynamic system (I) under different consumers sensitivity coefficient of carbon emissions

9. Discussions

The evolutionary behavior between duopoly manufacturers is various under the low carbon preference of consumers. We find that the evolutionary equilibrium results of the system is affected by the cost of reducing unit carbon emissions and the consumer sensitivity coefficient of carbon emissions. The government can encourage manufacturer to reduce emissions by implementing carbon taxes, and the carbon tax rate must be greater than boundary conditions. In addition, government can encourage manufacturer to reduce emissions by reducing the costs of reducing carbon emissions and increase consumer sensitivity coefficient of carbon emissions.

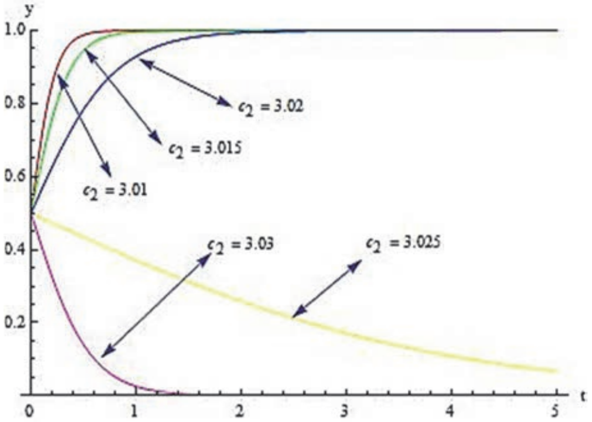

Fig. 4. Evolutionary path of the manufacturers' behavioral strategy in the replicator dynamic system (I) under different production cost of adopted low-carbon manufacturing

However, this paper faced some limitations. First, as we all know, consumer behavior is different, so low-carbon preferences of consumers is different. This paper considers the same situation of low carbon preferences for consumers. Second, the probability of manufacturers adopting low-carbon technology does not only depend on the incentive mechanism of the government, but also on other uncertainties. This paper only considers the game between the government and the manufacturer. Third, Carbon tax can promote manufacturers to reduce carbon emissions, but it may bring some other problems, which are uncertain

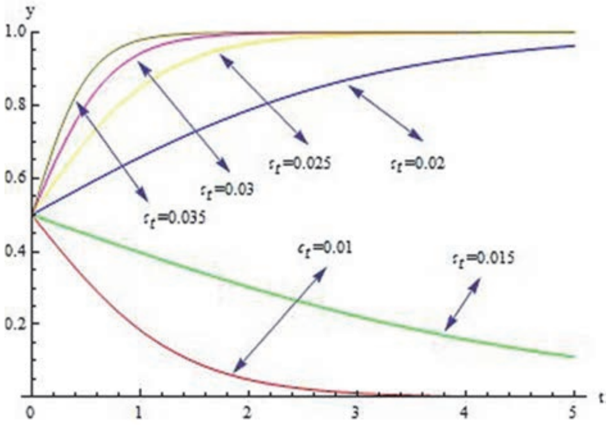

Fig. 5. Evolutionary path of the manufacturers' behavioral strategy in the replicator dynamic system (II) under different carbon tax rate

10. Conclusions

This paper considers low carbon preference of consumers, established the evolutionary game model to analyze the dynamic evolutionary process of manufacturer decision with reducing carbon emissions. The results show that the cost of reducing unit carbon emissions and the consumer sensitivity coefficient of carbon emissions are affect the evolutionary equilibrium results. There is boundary condition of the carbon tax rate, and only when the carbon tax rate is greater than boundary conditions, the carbon tax policy can effectively encourage manufacturers to reduce carbon emissions. In order to promote manufacturer should to reducing carbon emissions, government should reduce the costs of reducing carbon emissions and increase consumer sensitivity coefficient of carbon emissions, and the carbon tax rate must be greater than boundary condition.

In the future, we can study that analyze the decision of manufacturer about reducing carbon emissions and consider low carbon preference of consumers is different. In addition, this paper uses the carbon tax policy to promote the evolution of the system, how to use other carbon regulation policies to promote the evolution of the system will be study.

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